Planar Transformations

① $A_{(3,-5)}(2,7) = (2+3, 7-5) = (5,2)$

② $\begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3(-2)+2(5) \\ -1(-2)+4(5) \end{pmatrix} = \begin{pmatrix} 4 \\ 22 \end{pmatrix}$

③ $\begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 0 & -3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3(0)+2(1) & 3(-3)+2(2) \\ -1(0)+4(1) & -1(-3)+4(2) \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ 4 & 11 \end{pmatrix}$

④ $\begin{pmatrix} 2 & -5 \\ -1 & 6 \end{pmatrix}^{-1} = \dfrac{1}{2(6)-(-1)(-5)}\begin{pmatrix} 6 & 5 \\ 1 & 2 \end{pmatrix} = \dfrac{1}{7}\begin{pmatrix} 6 & 5 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 6/7 & 5/7 \\ 1/7 & 2/7 \end{pmatrix}$

Equations in One Variable

Circle the equations below that have no solution.

Ⓐ $x^2 = 4$

Ⓑ $e^{x^2-3} = 0$

An exponential of base e is always positive.

Ⓒ $2x-3 = x+7$

Ⓓ $\log_e(x)^3 = 2$

Ⓔ $(2x-1)^2 = 5$

Ⓕ $\log_e(x) = -12$

Ⓖ $\sqrt{2^x} = -4$

square roots can't be negative

Ⓗ $\sqrt[3]{2x-7} = -10$

Ⓘ $(3x^2-4)^2 = -1$

squares can't be negative

Quadratic Equations in $f(x)$

① Solve for x if $\frac{x^2}{2} + 2x - 3 = 0$.

$a = \frac{1}{2}$, $b = 2$, $c = -3$,

so $x = \frac{-2 \pm \sqrt{2^2 - 4(\frac{1}{2})(-3)}}{2(\frac{1}{2})} = -2 \pm \sqrt{10}$

That is, $x = -2 + \sqrt{10}$ or $x = -2 - \sqrt{10}$

② Solve for y if $\frac{y^2}{2} + 2y - 3 = 0$.

$a = \frac{1}{2}$, $b = 2$, $c = -3$,

so $y = -2 + \sqrt{10}$ or $y = -2 - \sqrt{10}$

③ Solve for $\log_e(x)$ if $\frac{\log_e(x)^2}{2} + 2\log_e(x) - 3 = 0$.

$a = \frac{1}{2}$, $b = 2$, $c = -3$,

so $\log_e(x) = -2 + \sqrt{10}$ or $\log_e(x) = -2 - \sqrt{10}$

④ Solve for x if $\frac{\log_e(x)^2}{2} + 2\log_e(x) - 3 = 0$.

(•) Implied domain is $(0, \infty)$ since we can only take the logarithm of a positive number.

(•) As in ③, we can use the quadratic formula to find that $\log_e(x) = -2 + \sqrt{10}$ or $\log_e(x) = -2 - \sqrt{10}$.
If $\log_e(x) = -2 + \sqrt{10}$, then $x = e^{-2+\sqrt{10}}$.
If $\log_e(x) = -2 - \sqrt{10}$, then $x = e^{-2-\sqrt{10}}$. Both of these solutions are in the domain, because they are positive.

$$\frac{\log_e(x)^2}{2} + 2\log_e(x) - 3 = 0 \begin{cases} \log_e(x) = -2 + \sqrt{10} \rightarrow x = e^{-2+\sqrt{10}} \\ \log_e(x) = -2 - \sqrt{10} \rightarrow x = e^{-2-\sqrt{10}} \end{cases}$$

Equations in Two Variables

① Write an equation for a line of slope -3 that passes through the origin.

$$y = -3x$$

② Write an equation for a line of slope 6 that passes through the point $(3,-5)$.

$$(y+5) = 6(x-3)$$

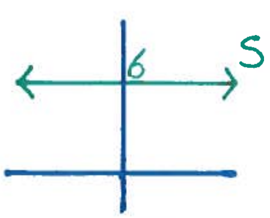
③ Find the slope of the line containing the points $(1,3)$ and $(7,-2)$.

$$\frac{3-(-2)}{1-7} = \frac{5}{-6}$$

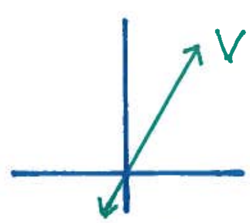
④ Write an equation for the line from problem 3.

There are two possible answers, using the point $(1,3)$ we have $(y-3) = -\frac{5}{6}(x-1)$. Using the point $(7,-2)$ we have $(y+2) = -\frac{5}{6}(x-7)$.

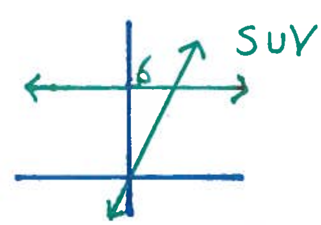
⑤ Write an equation for $S \cup V$ below.



$$y = 6$$
$$y - 6 = 0$$



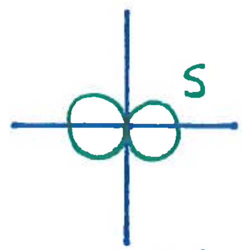
$$y = 2x$$
$$y - 2x = 0$$



$$(y-6)(y-2x) = 0$$

POTS

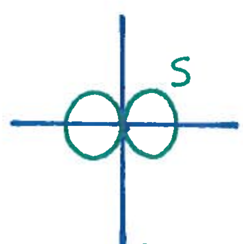
The "Lemniscate of Bernoulli" is the set of solutions, S, of the polynomial equation $[x^2+y^2]^2=x^2-y^2$



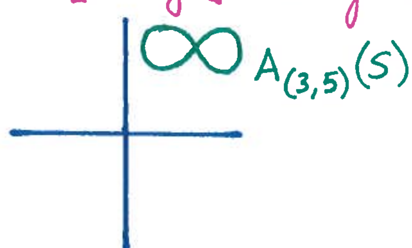
$$[x^2+y^2]^2=x^2-y^2$$

① Write an equation for $A_{(3,5)}(S)$.

(What's $A_{(3,5)}^{-1}$? What's $[x^2+y^2]^2=x^2-y^2$ composed with $A_{(3,5)}^{-1}$?)



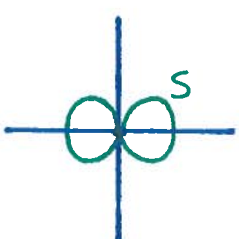
$$\xrightarrow{A_{(3,5)}}$$



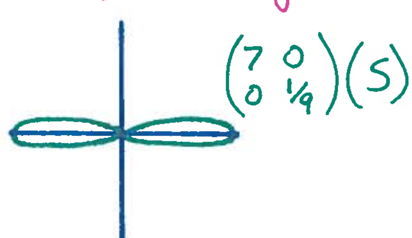
$$[x^2+y^2]^2=x^2-y^2 \xrightarrow[\substack{x\mapsto x-3\\ y\mapsto y-5}]{A_{(-3,-5)}} [(x-3)^2+(y-5)^2]^2=(x-3)^2-(y-5)^2$$

② Write an equation for $\begin{pmatrix} 7 & 0 \\ 0 & 1/9 \end{pmatrix}(S)$.

(What's $\begin{pmatrix} 7 & 0 \\ 0 & 1/9 \end{pmatrix}^{-1}$? What's $[x^2+y^2]^2=x^2-y^2$ composed with $\begin{pmatrix} 7 & 0 \\ 0 & 1/9 \end{pmatrix}$?)



$$\xrightarrow{\begin{pmatrix} 7 & 0 \\ 0 & 1/9 \end{pmatrix}}$$



$$[x^2+y^2]^2=x^2-y^2 \xrightarrow[\substack{x\mapsto \frac{x}{7}\\ y\mapsto 9y}]{\begin{pmatrix} 1/7 & 0 \\ 0 & 9 \end{pmatrix}} \left[\left(\frac{x}{7}\right)^2+(9y)^2\right]^2=\left(\frac{x}{7}\right)^2-(9y)^2$$