First Name:____________________ Last Name:____________________

1.) E

2.) J

3.) H

4.) C

5.) G

6.) H

7.) K

8.) B

9.) A

10.) D

11.) I

12.) F

13.) $\sqrt{34}$

14.) $\sqrt{41}$

15.) $\sin(\theta) = \frac{4}{5}$

$\cos(\theta) = \frac{3}{5}$

$\tan(\theta) = \frac{4}{3}$

16.) $\frac{7}{5\sqrt{2}}$

17.) $-\frac{1}{4}$

18.) $\sqrt{34-15\sqrt{3}}$

19.) $\sqrt{10}\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

20.) $7(\cos(-3), \sin(-3))$

21.) $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

22.) $\left(\frac{4}{\sqrt{2}}, \frac{10}{\sqrt{2}}\right)$

23.) $(y+2)^2 = (x-3)^3 + 3(x-3)^2$

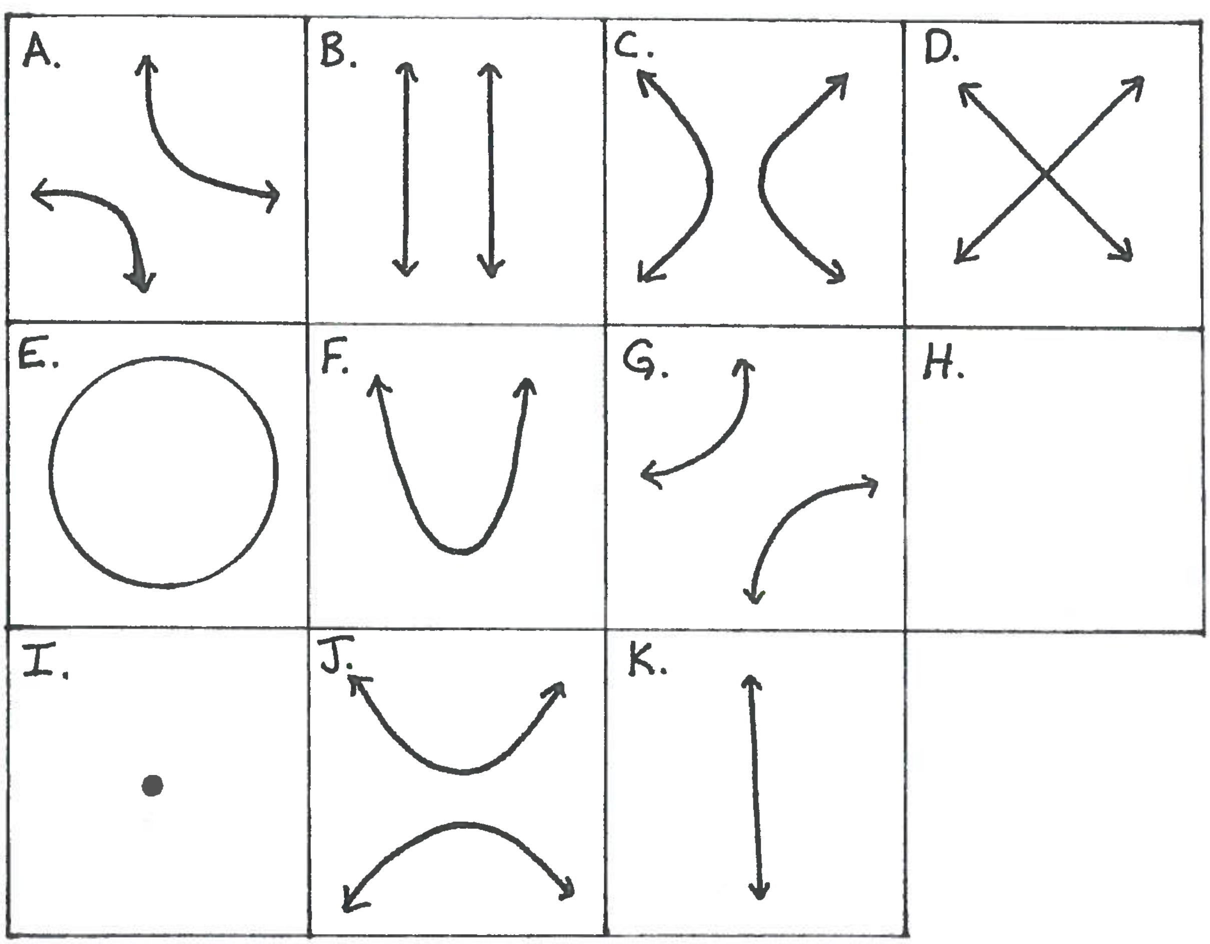
24.) $\left(\frac{y}{2}\right)^2 = (3x)^3 + 3(3x)^2$

Second Midterm Exam

Conics

For #1-12, match the numbered quadratic equations in two variables with their lettered sets of solutions. Worth $\frac{1}{2}$ point each.

1.) $x^2 + y^2 = 1$ E	2.) $y^2 - x^2 = 1$ J	3.) $x^2 = -1$ H
4.) $x^2 - y^2 = 1$ C	5.) $xy = -1$ G	6.) $x^2 + y^2 = -1$ H
7.) $x^2 = 0$ K	8.) $x^2 = 1$ B	9.) $xy = 1$ A
10.) $x^2 - y^2 = 0$ D	11.) $x^2 + y^2 = 0$ I	12.) $y = x^2$ F



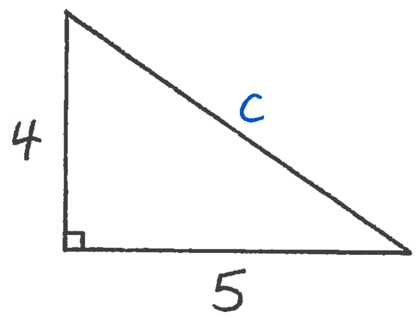
****

Trigonometry

13.) What is the distance between the points $(-2, 3)$ and $(1, -2)$?

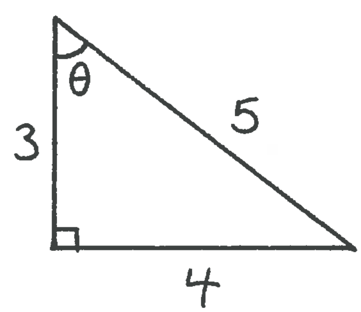
$$\sqrt{(-2-1)^2+(3-(-2))^2} = \sqrt{9+25} = \sqrt{34}$$

14.) Find the length of the unlabled side of the triangle below.



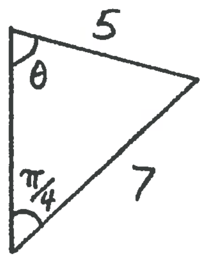
$$c^2 = 4^2 + 5^2$$
$$= 16 + 25$$
$$= 41$$
$$\Rightarrow$$
$$c = \sqrt{41}$$

15.) Find $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$ for the angle θ given below. (3 points.)



$$\sin(\theta) = \frac{4}{5}$$
$$\cos(\theta) = \frac{3}{5}$$
$$\tan(\theta) = \frac{4}{3}$$

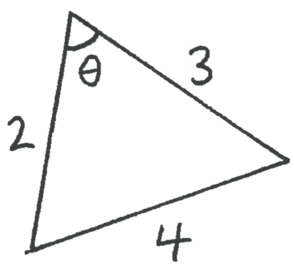
16.) Find $\sin(\theta)$ for the angle θ given below.



$$\frac{\sin(\theta)}{7} = \frac{\sin(\frac{\pi}{4})}{5}$$

$$\sin(\theta) = \frac{7\left(\frac{1}{\sqrt{2}}\right)}{5} = \frac{7}{5\sqrt{2}}$$

17.) Find $\cos(\theta)$ for the angle θ given below.



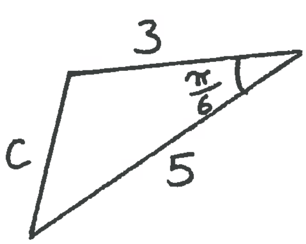
$$4^2 = 2^2 + 3^2 - 2(2)(3)\cos(\theta)$$

$$16 = \overset{13}{\cancel{}} - 12\cos(\theta)$$

$$3\cancel{} = -12\cos(\theta)$$

$$\cos(\theta) = -\frac{3}{12} = -\frac{1}{4}$$

18.) Find the length c shown below.



$$c^2 = 3^2 + 5^2 - 2(3)(5)\cos\left(\frac{\pi}{6}\right)$$

$$c^2 = 9 + 25 - 30\,\frac{\sqrt{3}}{2}$$

$$c^2 = 34 - 15\sqrt{3}$$

$$c = \sqrt{34 - 15\sqrt{3}}$$

19.) Write the vector $(3, -1)$ in polar coordinates.

$$\|(3,-1)\| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$$

$$(3,-1) = \sqrt{10}\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

20.) Rotate the point $7\big(\cos(3), \sin(3)\big)$ clockwise by an angle of 6.

$$7\big(\cos(3-6), \sin(3-6)\big) = 7\big(\cos(-3), \sin(-3)\big)$$

21.) Write the matrix that rotates the plane counterclockwise by an angle of $\frac{7\pi}{4}$. Simplify your answer so that it does not contain the letters sin or cos.

$$\begin{pmatrix} \cos\left(\frac{7\pi}{4}\right) & -\sin\left(\frac{7\pi}{4}\right) \\ \sin\left(\frac{7\pi}{4}\right) & \cos\left(\frac{7\pi}{4}\right) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

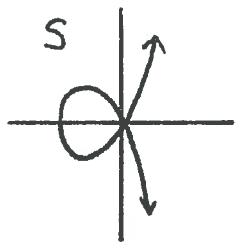
22.) The matrix that rotates the plane counterclockwise by an angle of $\frac{17\pi}{4}$ is $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$. Rotate the vector $(7, 3)$ counterclockwise by an angle of $\frac{17\pi}{4}$. Write your answer as a row vector.

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 7 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{7}{\sqrt{2}} - \frac{3}{\sqrt{2}} \\ \frac{7}{\sqrt{2}} + \frac{3}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{4}{\sqrt{2}} \\ \frac{10}{\sqrt{2}} \end{pmatrix} = \left(\frac{4}{\sqrt{2}}, \frac{10}{\sqrt{2}}\right)$$

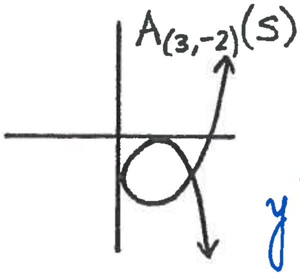
****

Transformations of Solutions of Equations in Two Variables

The "Tschirnhausen cubic" is the set of solutions, S, of the polynomial equation $y^2 = x^3 + 3x^2$.

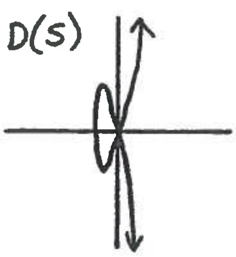


23.) Give an equation for $A_{(3,-2)}(S)$, the Tschirnhausen cubic shifted right 3 and down 2.



$$y^2 = x^3 + 3x^2 \xrightarrow[\substack{x \mapsto x-3 \\ y \mapsto y+2}]{A_{(3,-2)}^{-1}} (y+2)^2 = (x-3)^3 + 3(x-3)^2$$

24.) Let $D = \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & 2 \end{pmatrix}$. Give an equation for $D(S)$, the Tschirnhausen cubic scaled by $\frac{1}{3}$ in the x-coordinate and 2 in the y-coordinate.

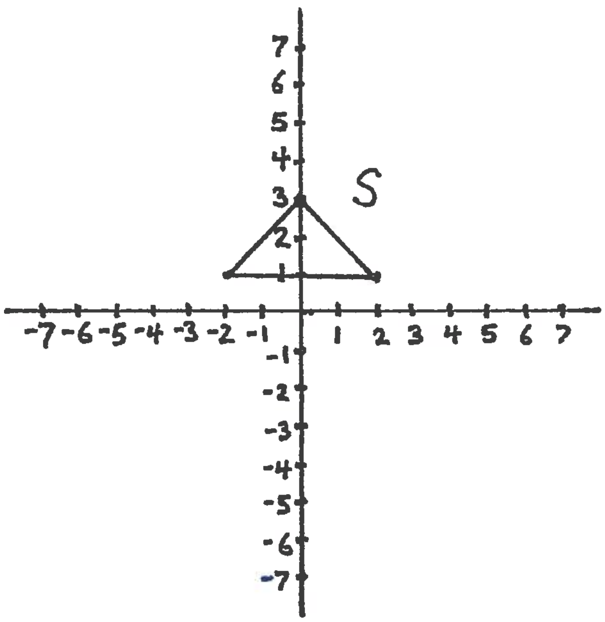


$$y^2 = x^3 + 3x^2 \xrightarrow[\substack{x \mapsto 3x \\ y \mapsto \frac{y}{2}}]{\begin{pmatrix} 3 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}} \left(\frac{y}{2}\right)^2 = (3x)^3 + 3(3x)^2$$

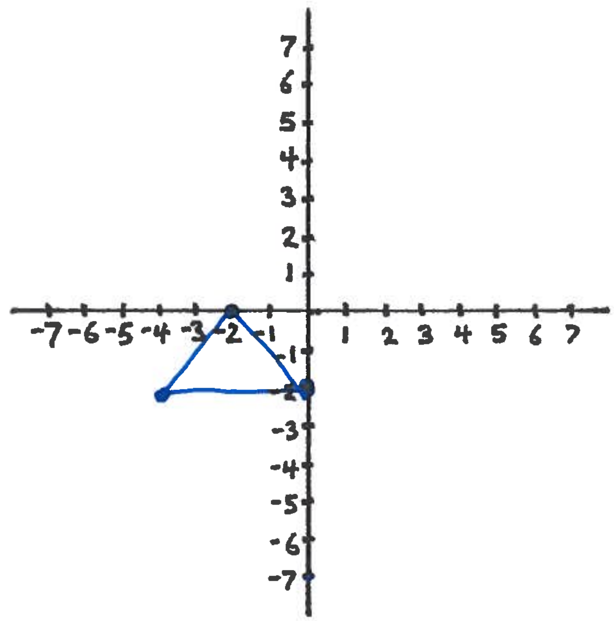
****

Planar Transformations

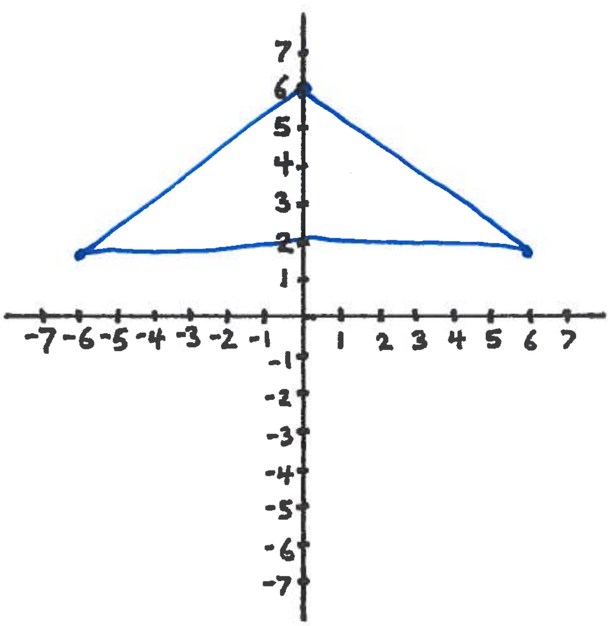
Shown below is a set S in the plane. (The x- and y-axes are not part of S. They are just drawn for perspective.)



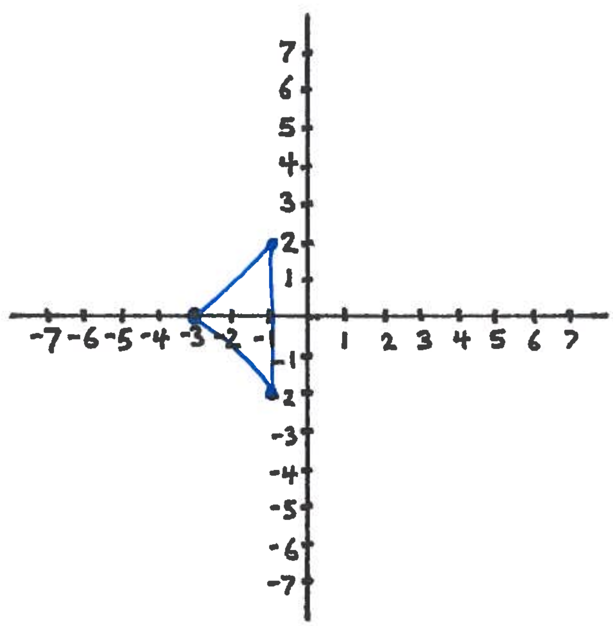
25.) Draw $A_{(-2,-3)}(S)$



26.) Draw $\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}(S)$



27.) Draw $R_{\frac{\pi}{2}}(S)$

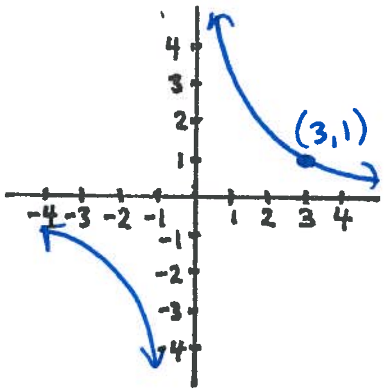


****

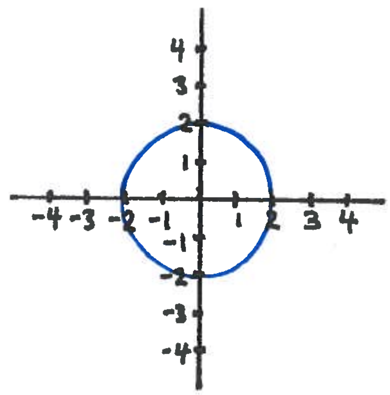
Conics

For #28-30, draw the set of solutions of the given equation in two variables. (Label at least one point precisely in #28.)

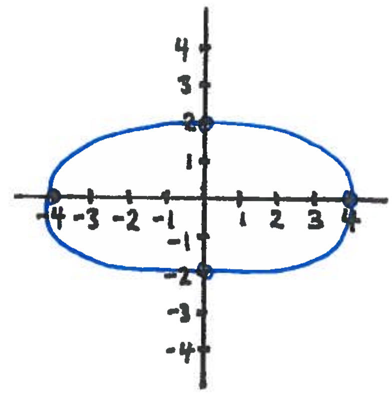
28.) $xy = 3$



29.) $x^2 + y^2 = 4$



30.) $\frac{x^2}{16} + \frac{y^2}{4} = 1$

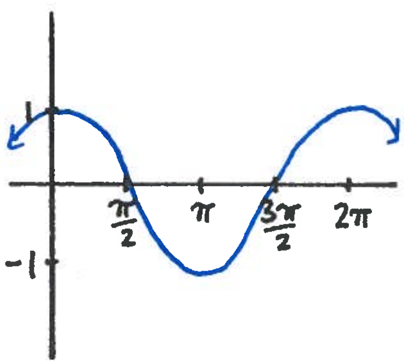


****

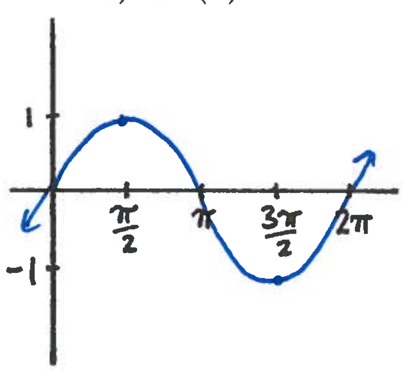
Trigonometric Functions

For #31-36, draw the graphs of the given functions.

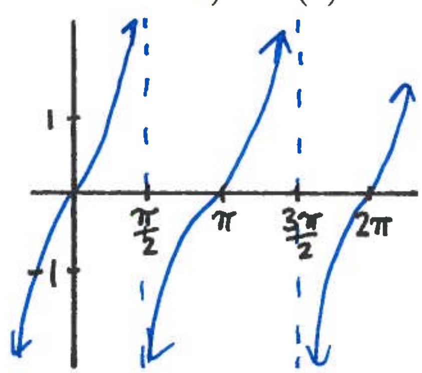
31.) $\cos(\theta)$



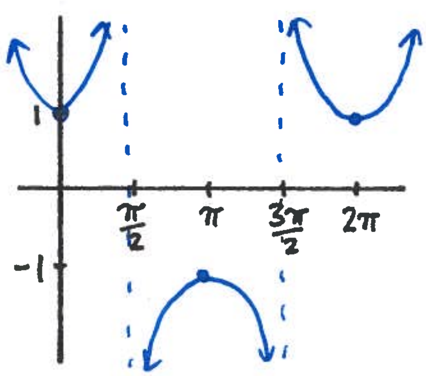
32.) $\sin(\theta)$



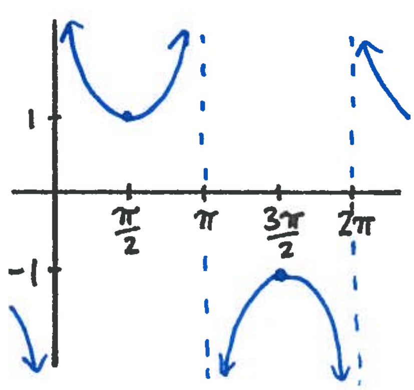
33.) $\tan(\theta)$



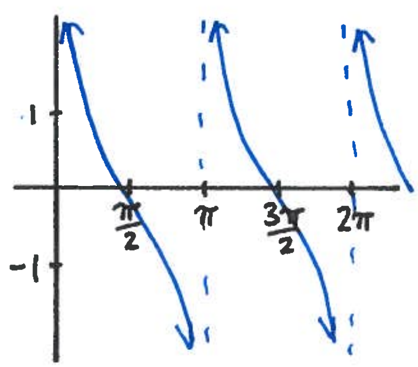
34.) $\sec(\theta)$



35.) $\csc(\theta)$



36.) $\cot(\theta)$



****

Equations in One Variable

The remaining questions are each worth 2 points. For #37-42, find the solutions of the given equations, and show your work. If an equation has no solution, explain why.

37.) $\log_2(4-x) = 3$ Domain: $4-x>0 \Rightarrow 4>x$

$4-x=2^3=8$

$4=x+8$

$\boxed{x=-4}$

38.) $(x-3)^2 = -1$

$\boxed{\text{No solution.}}$ Squares can't be negative

39.) $(x-3)^2 = 4$ Domain: $\mathbb{R}$

$x-3=2$ or $x-3=-2$

$\boxed{x=5 \quad \text{or} \quad x=1}$

40.) $\log_e(x) + \log_e(x+1) = 0$

Domain: $x > 0$ and $x + 1 > 0 \Rightarrow x > 0$ and $x > -1$ $\Rightarrow x > 0$

$$\log_e(x(x+1)) = 0$$

$$x(x+1) = e^0 = 1$$

$$x^2 + x = 1$$

$$x^2 + x - 1 = 0$$

Discriminant: $1^2 - 4(1)(-1) = 5$

$\Rightarrow$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

$$x = \frac{-1-\sqrt{5}}{2} \quad \text{or} \quad x = \frac{-1+\sqrt{5}}{2}$$

$\frac{-1-\sqrt{5}}{2} < 0$, so it is not in the domain, but $\sqrt{5} > 1$, so $-1+\sqrt{5} > 0$, and $\frac{-1+\sqrt{5}}{2} > 0$, so $\frac{-1+\sqrt{5}}{2}$ is in the domain.

$$\boxed{x = \frac{-1+\sqrt{5}}{2}}$$

41.) $e^{x-3}(e^{x+4})^5 = 2$

Domain: $\mathbb{R}$.

$$e^{x-3}e^{5(x+4)} = 2$$

$$e^{x-3+5(x+4)} = 2$$

$$e^{6x+17} = 2$$

$$6x + 17 = \log_e(2)$$

$$6x = \log_e(2) - 17$$

$$\boxed{x = \frac{\log_e(2) - 17}{6}}$$

42.) $\frac{\frac{x+1}{x}}{\frac{1}{x^2}} = -2$

Domain: $\mathbb{R} - \{0\}$ (can't divide by 0).

$$\frac{\frac{x+1}{x}}{\frac{1}{x^2}} = \frac{x+1}{x}\,\frac{x^2}{1} = (x+1)x = x^2 + x$$

so we have $x^2 + x = -2$ or $x^2 + x + 2 = 0$

Discriminant: $1^2 - 4(1)(2) = -7 < 0$

so there are $\boxed{\text{no solutions}}$