First Midterm Exam

Conics

For #1-8, match the numbered quadratic equations in two variables with their lettered sets of solutions.

1.) $y = x^2$ B

2.) $x^2 = 1$ D

3.) $x^2 = 0$ A

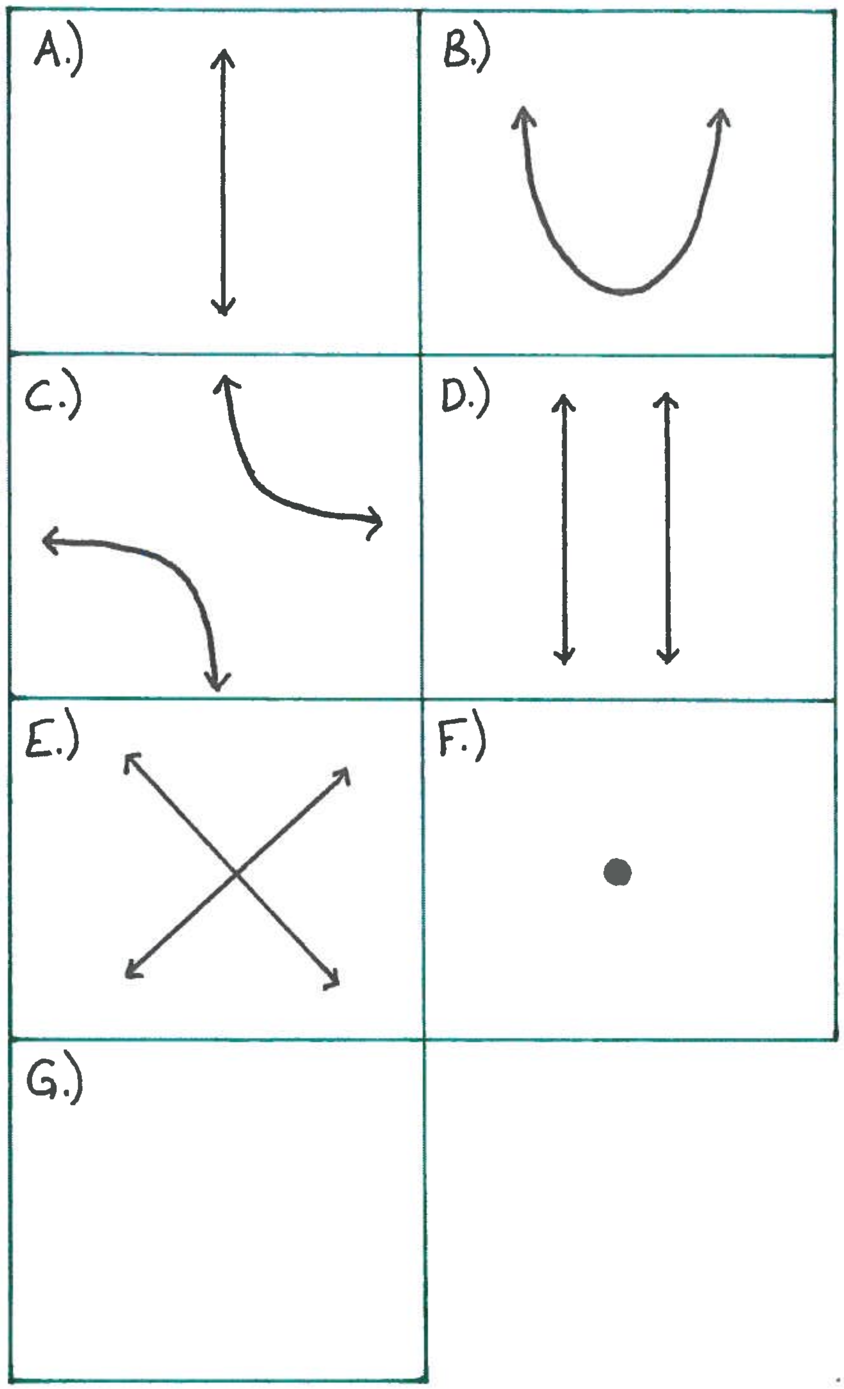
4.) $x^2 + y^2 = -1$ G

5.) $x^2 - y^2 = 0$ E

6.) $xy = 1$ C

7.) $x^2 = -1$ G

8.) $x^2 + y^2 = 0$ F



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Linear algebra

For #9-15, give the vector, written as a **ROW** vector.

9.) $A_{(2,-1)}(-4,6) = (-4+2, 6-1) = (-2,5)$

10.) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} = (3,5)$

11.) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ -8 \end{pmatrix} = \begin{pmatrix} -6 \\ -8 \end{pmatrix} = (-6,-8)$

12.) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \end{pmatrix} = (4,-7)$

13.) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 6 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} = (-1,6)$

14.) $\begin{pmatrix} 4 & 0 \\ 0 & 6 \end{pmatrix}\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4(-2) \\ 6(3) \end{pmatrix} = \begin{pmatrix} -8 \\ 18 \end{pmatrix} = (-8,18)$

15.) $\begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2(-2)+3(1) \\ 1(-2)-4(1) \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \end{pmatrix} = (-1,-6)$

16.) Find the product $\begin{pmatrix} 3 & 1 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} 2 & -4 \\ -1 & 2 \end{pmatrix}$

$$\begin{pmatrix} 3(2)+1(-1) & 3(-4)+1(2) \\ -2(2)-1(-1) & -2(-4)+(-1)(2) \end{pmatrix} = \begin{pmatrix} 5 & -10 \\ -3 & 6 \end{pmatrix}$$

17.) Give the determinant of $\begin{pmatrix} 4 & -3 \\ 1 & 2 \end{pmatrix}$

$$(4)(2)-(1)(-3) = 8+3=11$$

18.) Give the inverse of $\begin{pmatrix} -1 & 1 \\ -8 & 3 \end{pmatrix}$

$$\frac{1}{(-1)(3)-(-8)(1)} \begin{pmatrix} 3 & -1 \\ 8 & -1 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 3 & -1 \\ 8 & -1 \end{pmatrix} = \begin{pmatrix} 3/5 & -1/5 \\ 8/5 & -1/5 \end{pmatrix}$$

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Lines

19.) Give an equation for a line in the plane that has slope 4 and passes through the point $(0,0)$.

$$y=4x$$

20.) Give an equation for a line in the plane that has slope -2 and passes through the point $(6,1)$.

$$(y-1) = -2(x-6)$$

21.) Give the slope of the line that passes through the points $(3, 7)$ and $(5, 1)$.

$$\frac{7-1}{3-5} = \frac{6}{-2} = -3$$

22.) Give an equation for the line that passes through the points $(3, 7)$ and $(5, 1)$.

two possible answers:

$$(y-7) = -3(x-3)$$

or

$$(y-1) = -3(x-5)$$

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Equations in One Variable

23.) Give the implied domain of the equation $\frac{\log_e(x)^2}{2} - 3\log_e(x) + 2 = 0$.

$(0, \infty)$, because we can only take a logarithm of a numer if it's positive.

For #24-26, find the solutions of the given equations, and explain your answers. #24-26 are worth 2 points each.

24.) $\frac{\log_e(x)^2}{2} - 3\log_e(x) + 2 = 0$

Implied domain: $(0,\infty)$.

Quadratic equation in $\log_e(x)$ with $a=\frac{1}{2}$, $b=-3$, $c=2$, so $\log_e(x) = \dfrac{3 \pm \sqrt{9-4(\frac{1}{2})(2)}}{2(\frac{1}{2})} = 3 \pm \sqrt{5}$

If $\log_e(x) = 3+\sqrt{5}$, then $x = e^{3+\sqrt{5}}$. If $\log_e(x) = 3-\sqrt{5}$, then $x = e^{3-\sqrt{5}}$. Since $e^{3+\sqrt{5}}$ and $e^{3-\sqrt{5}}$ are positive, they are both solutions.

$$\frac{\log_e(x)^2}{2} - 3\log_e(x) + 2 = 0 \begin{cases} \rightarrow \log_e(x) = 3+\sqrt{5} \longrightarrow x = e^{3+\sqrt{5}} \\ \rightarrow \log_e(x) = 3-\sqrt{5} \longrightarrow x = e^{3-\sqrt{5}} \end{cases}$$

25.) $(4x-5)^2 = -3$

There's no solution because a square can never be negative.

26.) $\frac{x(x+2)}{x} = 2$

Implied domain is $\mathbb{R}-\{0\}$, because we can never divide by 0.

$$\frac{x(x+2)}{x} = 2 \Rightarrow (x+2) = 2 \Rightarrow x = 0$$

But 0 is not in the domain, so there is no solution of this equation.

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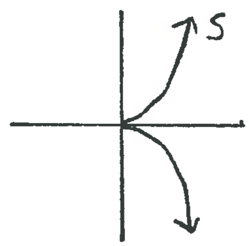
Equations in two variables and their solutions

27.) Suppose $p(x,y) = 3x^2 - 2xy + y - 3$. Find $p \circ A_{(-1,4)}(x,y)$. (You don't have to simplify your answer.)

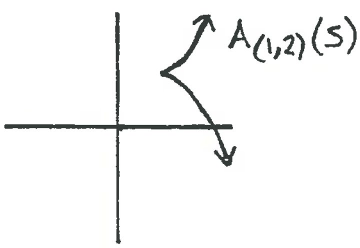
$$p \circ A_{(-1,4)}(x,y) = p(x-1, y+4)$$
$$= 3(x-1)^2 - 2(x-1)(y+4) + (y+4) - 3$$

#28-30 are worth 2 points each.

The "Cissoid of Diocles" is the set of solutions, S, of the polynomial equation $x^3 + xy^2 = y^2$.



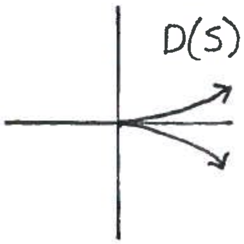
28.) Give an equation for $A_{(1,2)}(S)$, the Cissoid of Diocles shifted right 1 and up 2. (You don't have to simplify your answer.)



$$S \xrightarrow{A_{(1,2)}} A_{(1,2)}(S)$$

$$x^3 + xy^2 = y^2 \xrightarrow[\substack{x \mapsto x-1 \\ y \mapsto y-2}]{A_{(-1,-2)}} \boxed{(x-1)^3 + (x-1)(y-2)^2 = (y-2)^2}$$

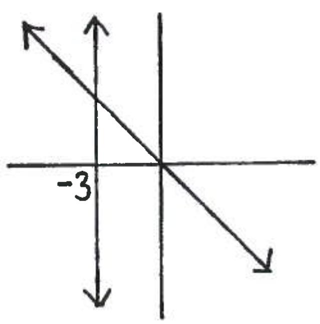
29.) Let $D = \begin{pmatrix} 9 & 0 \\ 0 & \frac{1}{5} \end{pmatrix}$. Give an equation for $D(S)$, the Cissoid of Diocles scaled by 9 in the x-coordinate and $\frac{1}{5}$ in the y-coordinate. (You don't have to simplify your answer.)



$$S \xrightarrow{\begin{pmatrix} 9 & 0 \\ 0 & \frac{1}{5} \end{pmatrix}} D(S)$$

$$x^3 + xy^2 = y^2 \xrightarrow[\substack{x \mapsto \frac{x}{9} \\ y \mapsto 5y}]{\begin{pmatrix} \frac{1}{9} & 0 \\ 0 & 5 \end{pmatrix}} \boxed{\left(\frac{x}{9}\right)^3 + \left(\frac{x}{9}\right)(5y)^2 = (5y)^2}$$

30.) Give an equation whose set of solutions is the union of the line $y = -x$ and the line $x = -3$. (You don't have to simplify your answer.)



$y = -x$ is equivalent to $y + x = 0$.

$x = -3$ is equivalent to $x + 3 = 0$.

Therefore, the equation for the union of the two lines is

$$(y + x)(x + 3) = 0.$$