First Name:________________ Last Name:________________

1.) 1

2.) 1

3.) ∞

4.) 0

5.) 2

6.) 0

7.) 0

8.) 0

9.) 1

10.) 0

11.) 1

12.) 0

13.) ∞

14.) ∞

15.) 1

16.) $(-8,7)$

17.) $\begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix}$

18.) $y=-3x$

19.) $(y-5)=-3(x-1)$

20.) $x^2+y^2=1$

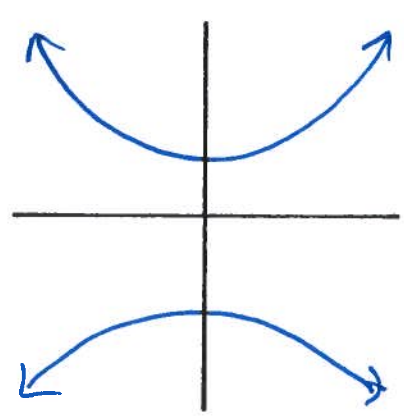
21.) $x^2+y^2=9$

22.) $(x-6)^2+(y-4)^2=9$

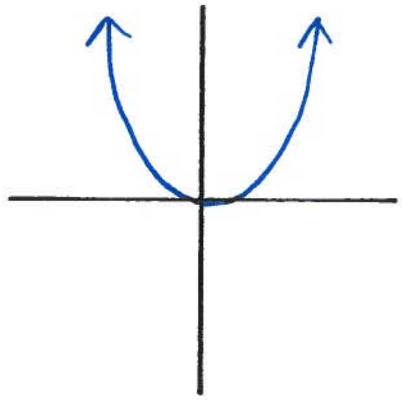
23.) $\frac{x^2}{4}+\frac{y^2}{16}=1$

24.) $\frac{(x-3)^2}{4}+\frac{(y-4)^2}{16}=1$

25.)



26.)



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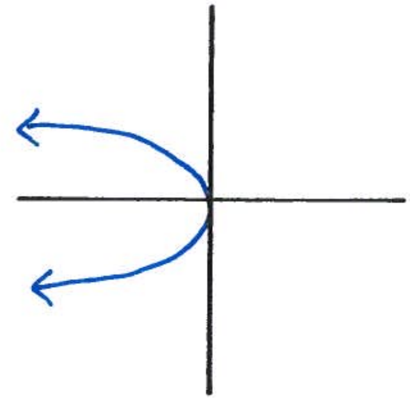

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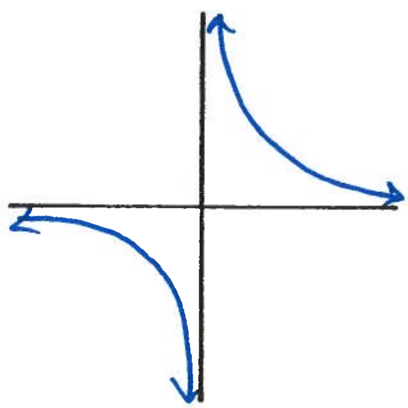
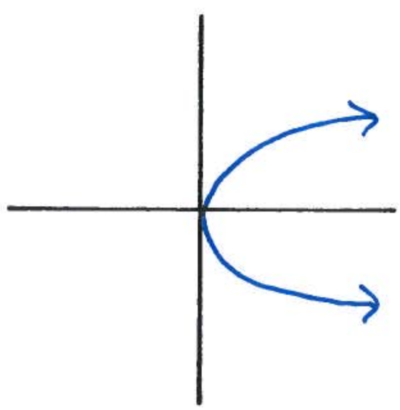


31.) $(x-3)(y-5)=1$

32.) ellipses, hyperbolas, and parabolas

33.) $\sqrt{13}$

28.)



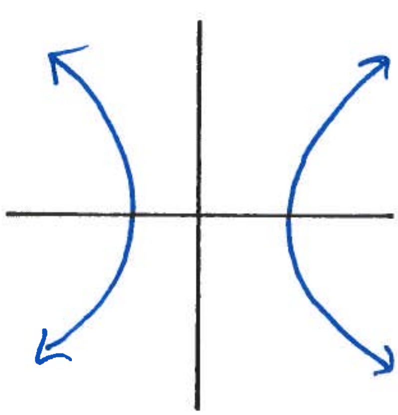
34.) $\sqrt{20}$

35.) $\sin(\theta) = \frac{5}{13}$

$\cos(\theta) = \frac{12}{13}$

$\tan(\theta) = \frac{5}{12}$

29.)



36.) A

37.) C

38.) A

39.) H

40.) E

41.) D

42.) J

43.) E

44.) F

45.) B

46.) G

47.) I

48.) B

49.) A

50.) $\pi/4$

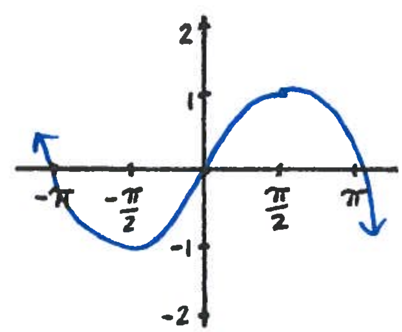
51.) $\pi/3$

52.) $\pi/3$

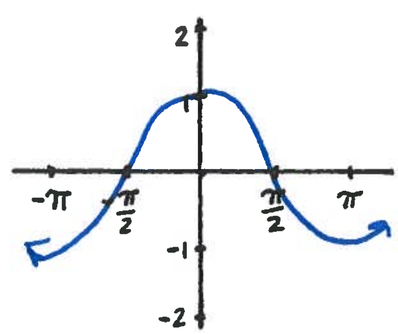
53.) $\cos(\theta)^2 + \sin(\theta)^2 = 1$

****

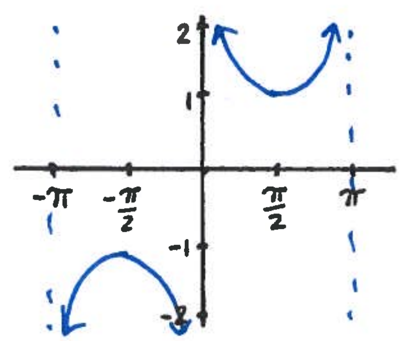
54.) $\sin(\theta)$



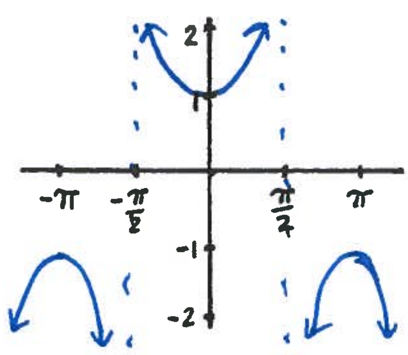
55.) $\cos(\theta)$



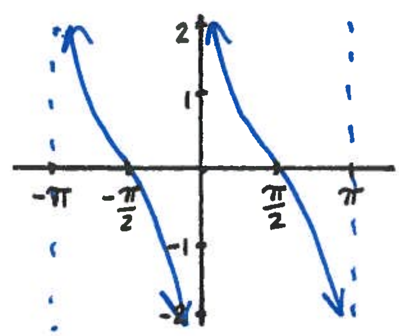
56.) $\csc(\theta)$



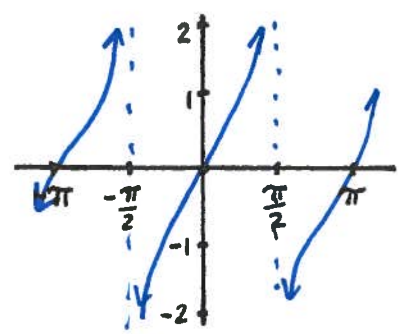
57.) $\sec(\theta)$



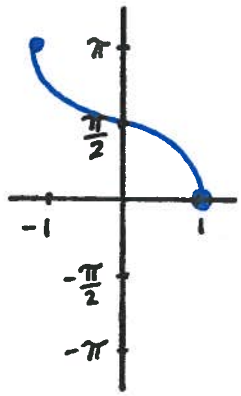
58.) $\cot(\theta)$



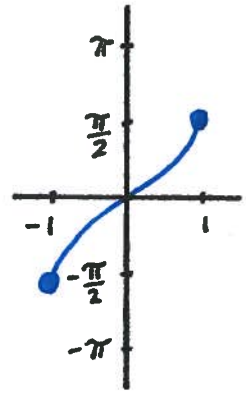
59.) $\tan(\theta)$



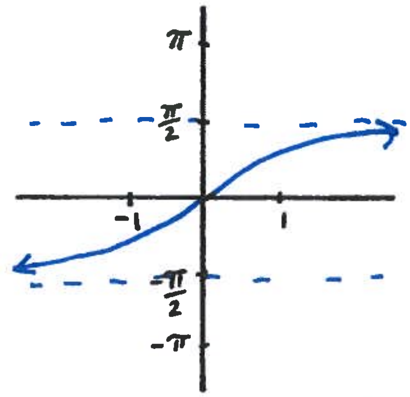
60.) $\arccos(\theta)$



61.) $\arcsin(\theta)$



62.) $\arctan(\theta)$



63.) $5+i8$

64.) $2+i11$

65.) $\sqrt{34}$

66.) $\sqrt{10}\left(\frac{3}{\sqrt{10}}+i\frac{1}{\sqrt{10}}\right)$

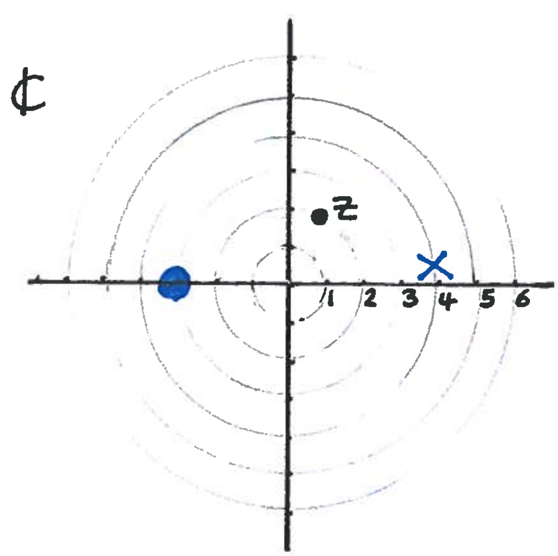
67.) $15\left(\cos(6)+i\sin(6)\right)$

68.) $-i$

69.) -1

****

70.) and 71.)



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Equations in one variable

For #72-77, give the real number solutions of the equations. If there is no solution, explain why there is no solution. For at least one of the problems, the domain of the equation will play an important role. Worth 2 points each.

72.) $\sqrt{3x-4} = -9$

no solutions, a square can't be negative.

73.) $\log_e(x) + \log_e(x-1) = 0$

Domain: $x>0$ and $x-1>0 \Rightarrow x>0$ and $x>1 \Rightarrow x>1$.

$\log_e(x(x-1))=0$

$x(x-1)=e^0=1$

$x^2-x-1=0$

$a=1, b=-1, c=-1$

$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$

since $\sqrt{5} > 1$, $\frac{1+\sqrt{5}}{2} > 1$,

so $\frac{1+\sqrt{5}}{2}$ is a solution.

However, $\frac{1-\sqrt{5}}{2} < 0$, so $\frac{1-\sqrt{5}}{2}$ is not a solution.

74.) $(x-17)^2 = 9$ Domain: $\mathbb{R}$

$x-17=3 \rightarrow x=20$

$x-17=-3 \rightarrow x=14$

75.) $\log_e(x)^2 + 3\log_e(x) + 1 = 0$ Domain: $x > 0$.

$a=1,\ b=3,\ c=1.$

$$\log_e(x) = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2} = \frac{-3 \pm \sqrt{5}}{2}$$

$$\log_e(x) = \frac{-3+\sqrt{5}}{2} \longrightarrow x = e^{\frac{-3+\sqrt{5}}{2}}$$

$$\log_e(x) = \frac{-3-\sqrt{5}}{2} \longrightarrow x = e^{\frac{-3-\sqrt{5}}{2}}$$

76.) $(x^2-5) = (x^2-5)x^3$ Domain: $\mathbb{R}$

$$x^2 - 5 = 0 \longrightarrow x^2 = 5 \longrightarrow x = \sqrt{5},\ x = -\sqrt{5}$$

$$1 = x^3 \longrightarrow x = 1.$$

77.) $e^{x+3}e^{x+2} = 1$ Domain: $\mathbb{R}$.

$$e^{2x+5} = 1$$

$$2x+5 = \log_e(1) = 0.$$

$$2x+5 = 0$$

$$2x = -5$$

$$x = \frac{-5}{2}$$

Final Exam

Equations in One Variable

How many real number solutions are there for the following equations? Your answers will either be 0, 1, 2, or ∞, if there are infinitely many solutions. Worth $\frac{1}{2}$ point each.

1.) $e^x = 2$ 1

2.) $x^2 = 0$ 1

3.) $\sin(x) = -\frac{4}{5}$ ∞

4.) $\sqrt{x} = -2$ 0

5.) $x^2 = 3$ 2

6.) $\cos(x) = 5$ 0

7.) $x^2 + x + 4 = 0$ 0

$1^2 - 4(1)4 = -15 < 0$

8.) $x^2 = -4$ 0

9.) $\log_e(x) = -1$ 1

10.) $e^x = 0$ 0

11.) $x^2 - 2x + 1 = 0$ 1

$(-2)^2 - 4(1)(1) = 0$

12.) $\sin(x) = -2$ 0

13.) $\tan(x) = -5$ ∞

14.) $\cos(x) = \frac{2}{3}$ ∞

15.) $\sqrt{x} = 4$ 1

****

Linear Algebra

16.) Give the vector written as a ROW vector $\begin{pmatrix} 3 & -2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 3(-2) + (-2)(1) \\ -3(-2) + 1(1) \end{pmatrix} = \begin{pmatrix} -6-2 \\ 6+1 \end{pmatrix} = \begin{pmatrix} -8 \\ 7 \end{pmatrix} = (-8, 7)$$

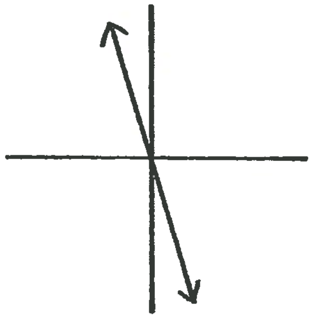
17.) Find the product $\begin{pmatrix} 0 & -3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 0 & -3 \end{pmatrix}$

$$\begin{pmatrix} 0 \cdot 5 + (-3) \cdot 0 & 2 \cdot 0 + (-3)(-3) \\ 1 \cdot 5 + 2 \cdot 0 & 2 \cdot 1 + 2(-3) \end{pmatrix} = \begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix}$$

****

Conics and other solutions of equations in two variables

18.) Give the equation for a line of slope -3 that passes through the origin.

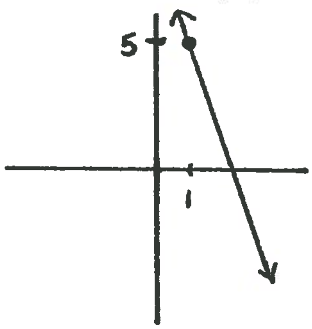


$y = -3x$

19.) Give the equation for a line of slope -3 that passes through the point $(1, 5)$.

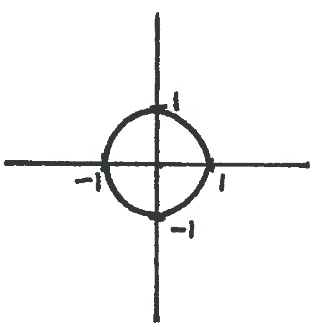
$x \mapsto x-1$

$y \mapsto y-5$



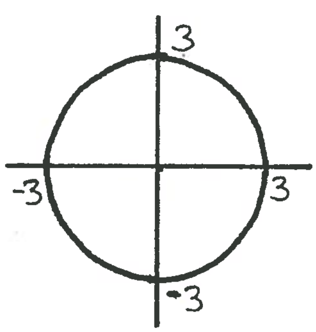
$(y-5) = -3(x-1)$

20.) Give the equation of the unit circle.



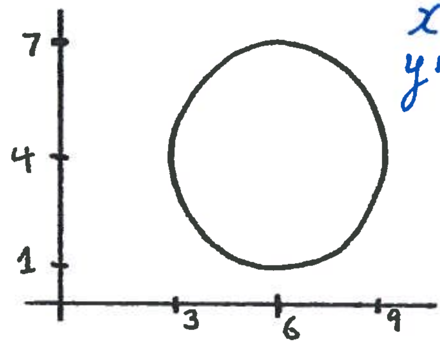
$x^2 + y^2 = 1$

21.) Give the equation of the circle of radius 3 centered at the origin.



$x^2 + y^2 = 9$

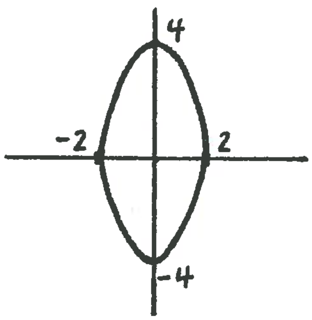
22.) Give the equation of the circle of radius 3 centered at the point $(6, 4)$.



$x \mapsto x-6$
$y \mapsto y-4$

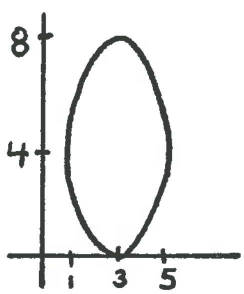
$$(x-6)^2 + (y-4)^2 = 9$$

23.) Give the equation of the ellipse obtained by starting with the unit circle, and then scaling the x-axis by 2, and the y-axis by 4.



$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

24.) Give the equation of the ellipse from #23, shifted right by 3 and up by 4.



$$\frac{(x-3)^2}{4} + \frac{(y-4)^2}{16} = 1$$

$x \mapsto x-3$
$y \mapsto y-4$

25.) Draw the set of solutions of the equation $y^2 - x^2 = 1$.

26.) Draw the set of solutions of the equation $y = x^2$.

27.) Draw the set of solutions of the equation $xy = 1$.

28.) Draw the set of solutions of the equation $x = y^2$.

29.) Draw the set of solutions of the equation $x^2 - y^2 = 1$.

30.) $R_{\frac{\pi}{2}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is the rotation of the plane by angle $\frac{\pi}{2}$. If $P \subseteq \mathbb{R}^2$ is the set of solutions of $y = x^2$ from #26, then draw $R_{\frac{\pi}{2}}(P)$.

31.) Let H be the set of solutions of $xy = 1$ from #27. What's the equation for H shifted right by 3 and up by 5?

$x \longmapsto x-3$

$y \longmapsto y-5$

$(x-3)(y-5)=1$

32.) Name the three types nondegenerate conics.

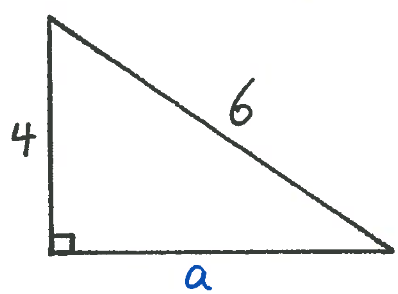
****

Trigonometry

33.) What is the distance between the points $(3, 4)$ and $(5, 1)$?

$\sqrt{(5-3)^2+(4-1)^2} = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$

34.) Find the length of the unlabeled side of the triangle below.



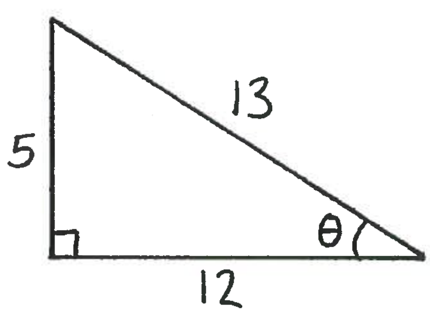
$6^2 = 4^2 + a^2$

$36 = 16 + a^2$

$a^2 = 20$

$a = \sqrt{20}$

35.) Find $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$ for the angle θ given below. (3 points.)



$\sin(\theta) = \frac{5}{13}$

$\cos(\theta) = \frac{12}{13}$

$\tan(\theta) = \frac{5}{12}$

Match the functions with their graphs.

36.) $\cos(x)$ A

39.) $\cos\left(\frac{x}{2}\right)$ H

42.) $\cos(x) + 1$ J

45.) $\cos\left(x + \frac{\pi}{2}\right)$ B

37.) $\frac{1}{2}\cos(x)$ C

40.) $\sin(x)$ E

43.) $\cos\left(x - \frac{\pi}{2}\right)$ E

46.) $\cos(2x)$ G

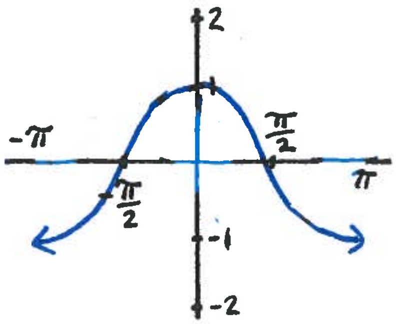
38.) $\cos(-x)$ A

41.) $\cos(x) - 1$ D

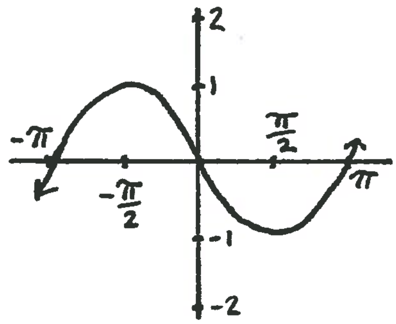
44.) $2\cos(x)$ F

47.) $-\cos(x)$ I

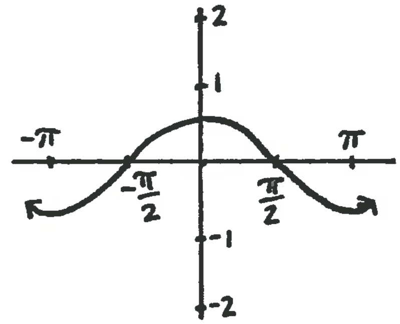
A.)



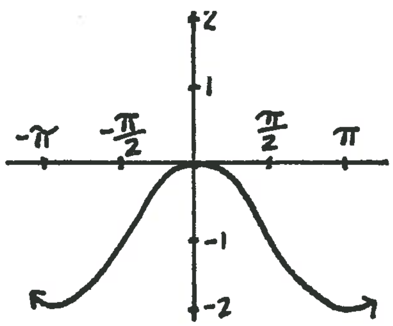
B)



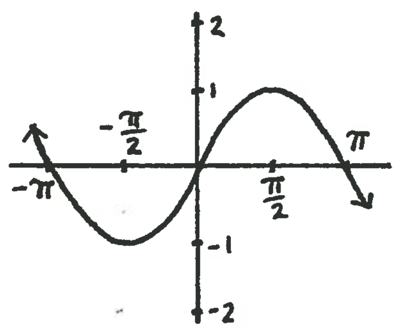
C.)



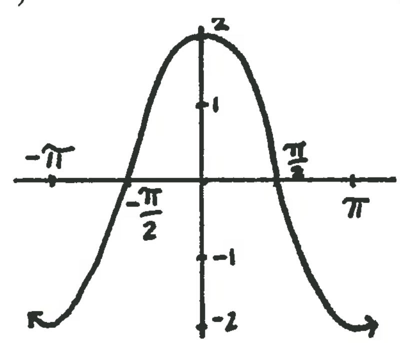
D.)



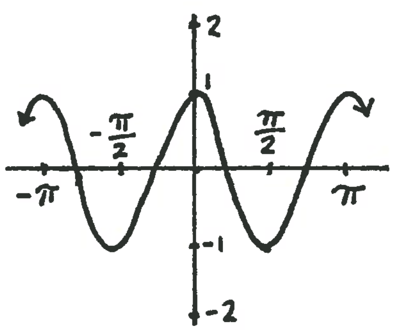
E.)



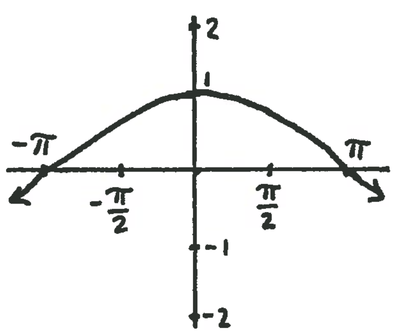
F.)



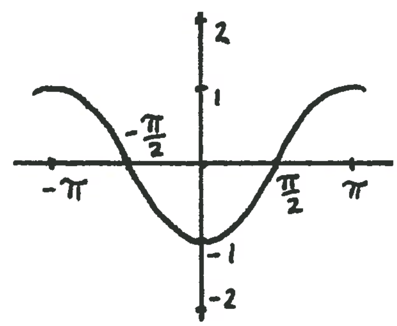
G.)



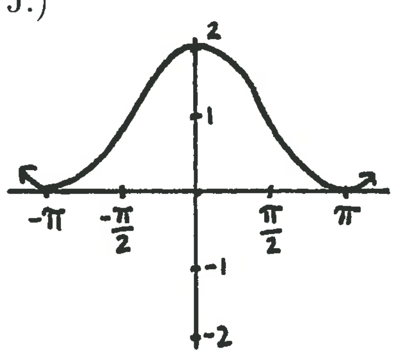
H.)



I.)



J.)

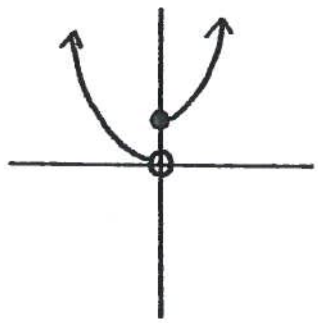


Match the functions with their graphs.

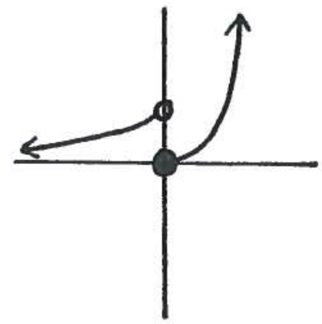
48.) $f(x) = \begin{cases} e^x & \text{if } x < 0; \\ x^2 & \text{if } x \geq 0. \end{cases}$ B

49.) $g(x) = \begin{cases} x^2 & \text{if } x < 0; \\ e^x & \text{if } x \geq 0. \end{cases}$ A

A.)



B.)



Find the values for #50-52. All answers will be either $\frac{\pi}{6}$ or $\frac{\pi}{4}$ or $\frac{\pi}{3}$.

50.) $\arccos\left(\frac{1}{\sqrt{2}}\right) = \pi/4$ since $\cos(\pi/4) = 1/\sqrt{2}$

51.) $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ since $\sin(\pi/3) = \frac{\sqrt{3}}{2}$

52.) $\arctan(\sqrt{3}) = \pi/3$ since $\tan(\pi/3) = \frac{\sin(\pi/3)}{\cos(\pi/3)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

53.) What's the Pythagorean Identity?

For #54-62, graph $\sin(x)$, $\cos(x)$, $\csc(x)$, $\sec(x)$, $\cot(x)$, $\tan(x)$, $\arcsin(x)$, $\arccos(x)$, and $\arctan(x)$.

****

Complex numbers

63.) Find $(3 + i4) + (2 + i4)$.

$$(3+2) + i(4+4) = 5 + i8$$

64.) Find $(3 + i4)(2 + i)$.

$$3 \cdot 2 + 3i + i4 \cdot 2 + i4i = 6 + i3 + i8 + i^2 4$$

$$= 6 - 4 + i(3+8)$$

$$= 2 + i11$$

65.) What's the norm of $3 + i5$.

$$|3+i5| = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$$

66.) Write $3 + i$ in polar coordinates.

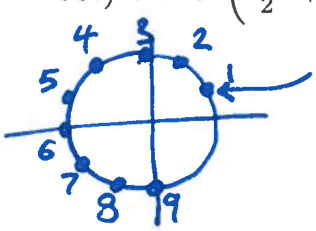
$|3+i| = \sqrt{3^2+1^2} = \sqrt{9+1} = \sqrt{10}$, so

$$3+i = \sqrt{10}\left(\frac{3}{\sqrt{10}} + i\frac{1}{\sqrt{10}}\right)$$

67.) Find $3(\cos(2) + i\sin(2))5(\cos(4) + i\sin(4))$.

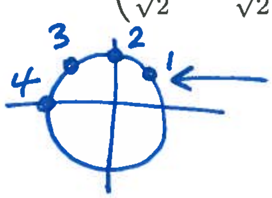
$3 \cdot 5 = 15$ and $2+4=6$, so $15(\cos(6)+i\sin(6))$

68.) Find $\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)^9 = -i$



$\frac{\sqrt{3}}{2} + i\frac{1}{2}$

69.) Find $\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)^4 = -1$



$\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$

70.) Draw a dot on the number $3\big(\cos(\pi) + i\sin(\pi)\big)$ norm 3, angle π.

71.) Draw an X on the number $2\big(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\big)z$.
(The number z is drawn on the answer sheet.)

scale by 2, rotate clockwise by $\pi/3$.