Planar Transformations

$$A_{(a,b)}(x,y) = (x+a, y+b)$$

$$x \longmapsto x+a$$

$$y \longmapsto y+b$$

$$\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ dy \end{pmatrix}$$

$$x \longmapsto ax$$

$$y \longmapsto dy$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$

$$x \longmapsto y$$

$$y \longmapsto x$$

$$A_{(a,b)}^{-1} = A_{(-a,-b)}$$

$$\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}^{-1} = \begin{pmatrix} 1/a & 0 \\ 0 & 1/d \end{pmatrix}$$

(if $a \neq 0$ and $d \neq 0$)

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

(if $ad-bc \neq 0$)

Equations in One Variable

Equivalent equations:

① $f(x)+h(x)=g(x)$ and $f(x)=g(x)-h(x)$

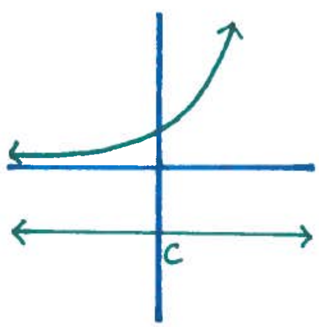
② $h(x)f(x)=g(x)$ and $f(x)=\frac{g(x)}{h(x)}$

(if $h(x)$ has no zeros in the domain)

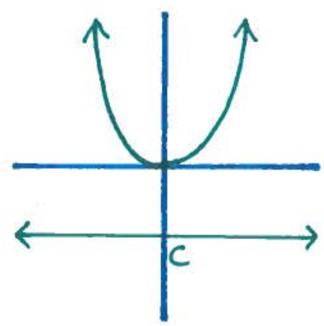
③ $h(f(x))=g(x)$ and $f(x)=h^{-1}(g(x))$

Equations with no solutions:

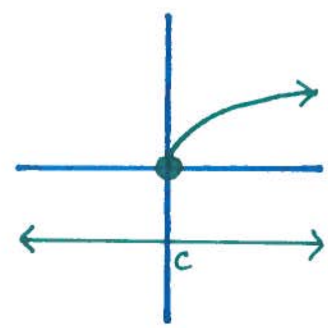
① $e^{f(x)}=c$ if $c\leq 0$



② $f(x)^2=c$ if $c<0$



③ $\sqrt{f(x)}=c$ if $c<0$

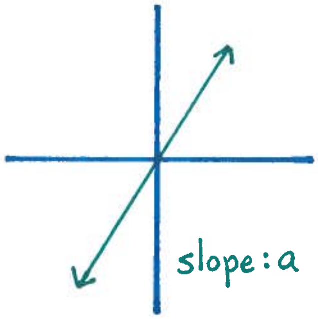


Quadratic equations:

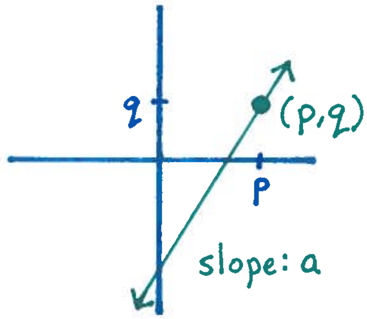
(•) If $ax^2+bx+c=0$, and $a\neq 0$, then $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

(•) If $af(x)^2+bf(x)+c=0$, and $a\neq 0$, then $f(x)=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

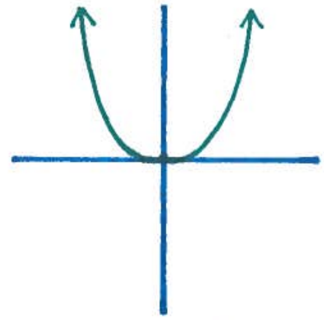
Equations in Two Variables



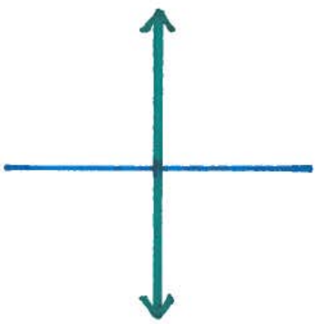
$y = ax$



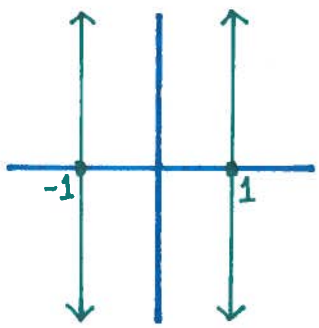
$(y-q) = a(x-p)$



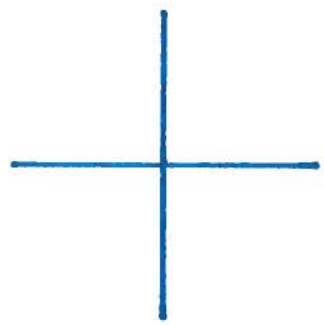
$y = x^2$



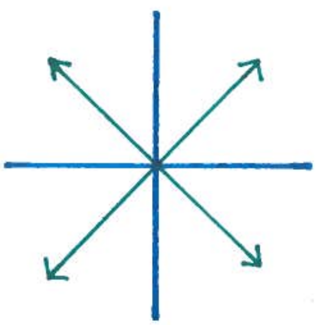
$x = 0$



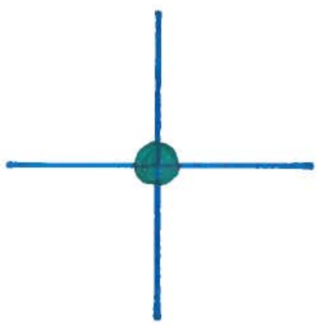
$x^2 = 1$



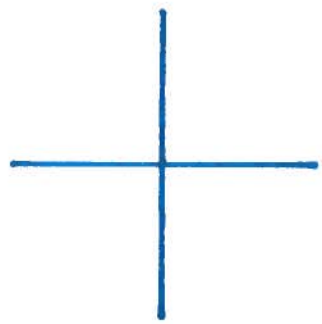
$x^2 = -1$



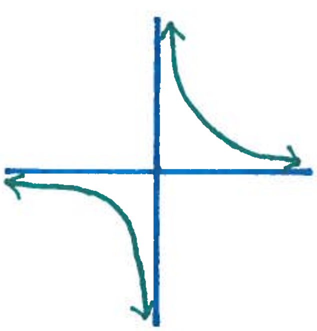
$x^2 - y^2 = 0$



$x^2 + y^2 = 0$

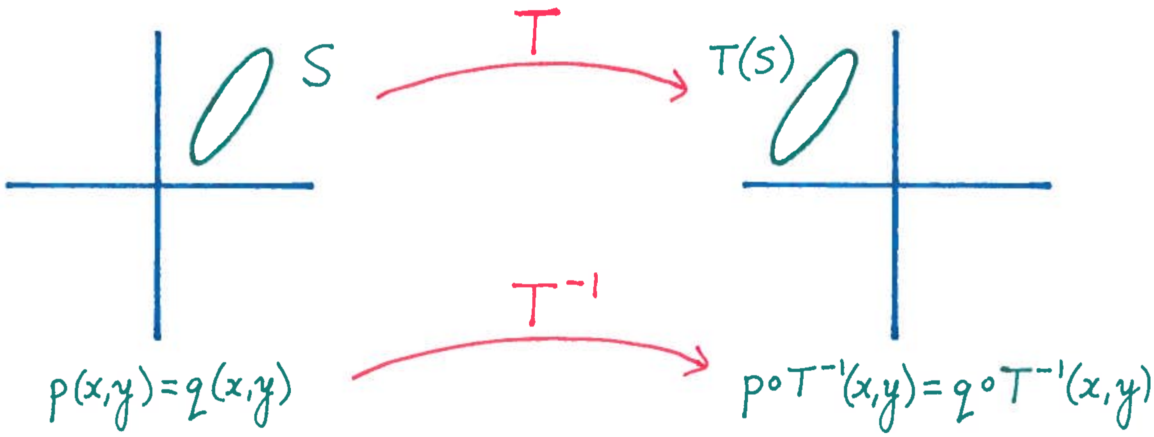


$x^2 + y^2 = -1$



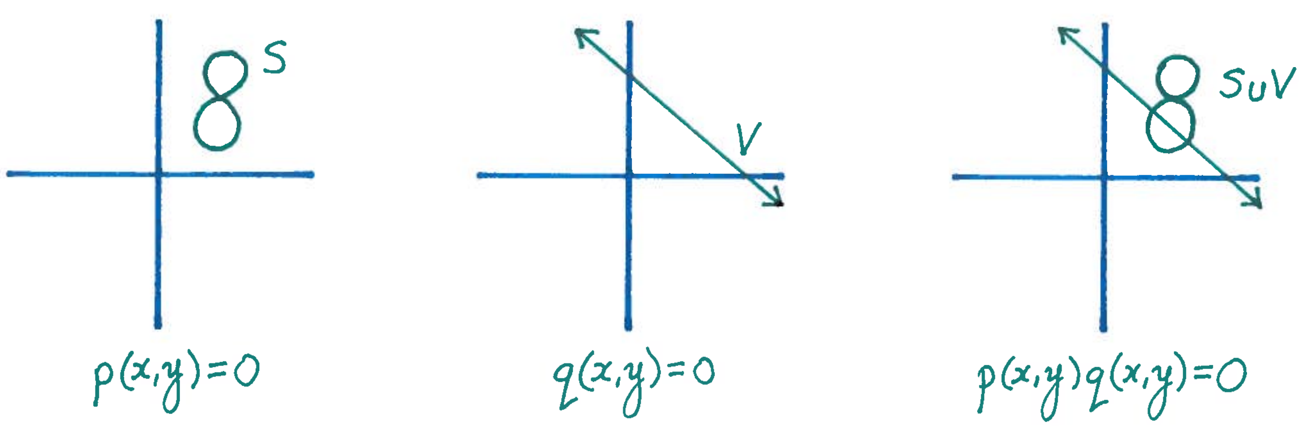
$xy = 1$

POTS:



S
T
$T(S)$
T^{-1}
$p(x,y)=q(x,y)$
$p \circ T^{-1}(x,y)=q \circ T^{-1}(x,y)$

Unions:



S
V
$S \cup V$
$p(x,y)=0$
$q(x,y)=0$
$p(x,y)q(x,y)=0$