# Vectors & Scalars

## Vectors

$\mathbb{R}^2$ is the set of all pairs of real numbers. In the context of drawing graphs, the objects in $\mathbb{R}^2$ are called points, and pairs are written left-to-right, so that $(3, 2)$ is the point in $\mathbb{R}^2$ whose $x$-coordinate equals 3 and whose $y$-coordinate equals 2.

In the context of linear algebra, the objects in $\mathbb{R}^2$ are called *vectors*, and instead of being written left-to-right, they are usually written top-to-bottom. Written in this way, the vector in $\mathbb{R}^2$ whose $x$-coordinate is 3 and whose $y$-coordinate is 2 is

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$\mathbb{R}^3$ is the set of all "triples" of real numbers. An object in $\mathbb{R}^3$ – also called a vector – has an $x$-coordinate, a $y$-coordinate, and a $z$-coordinate. When writing vectors in $\mathbb{R}^3$, the $x$-coordinate is on top, the $y$-coordinate is directly below, and the $z$-coordinate is on the bottom. Thus

$$\begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$$

is the vector in $\mathbb{R}^3$ where $x = 5$, $y = 0$, and $z = -1$.

## Vector addition

To add two vectors in $\mathbb{R}^2$ – or two vectors in $\mathbb{R}^3$ – add each of their coordinates.

**Examples.**

$$\begin{pmatrix} -5 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -5+4 \\ 1+2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

and

$$\begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} + \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} = \begin{pmatrix} 4+3 \\ 2-8 \\ 6+0 \end{pmatrix} = \begin{pmatrix} 7 \\ -6 \\ 6 \end{pmatrix}$$

## Scalar multiplication

In linear algebra, real numbers are often called *scalars*. You cannot multiply two vectors, but you can multiply a scalar and a vector. To do so, multiply every coordinate in the vector by the scalar.

**Examples.**

$$2\begin{pmatrix} 7 \\ -3 \end{pmatrix} = \begin{pmatrix} 2(7) \\ 2(-3) \end{pmatrix} = \begin{pmatrix} 14 \\ -6 \end{pmatrix}$$

and

$$5\begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 5(-1) \\ 5(0) \\ 5(4) \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 20 \end{pmatrix}$$

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# Exercises

For #1-8, perform the vector arithmetic indicated.

1.) $\begin{pmatrix} -5 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

2.) $\begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} + \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix}$

3.) $\begin{pmatrix} 2 \\ 8 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

4.) $\begin{pmatrix} 3 \\ 5 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix}$

5.) $2\begin{pmatrix} 7 \\ -3 \end{pmatrix}$

6.) $5\begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$

7.) $-1\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

8.) $3\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$

Completely factor the cubic polynomials below.

9.) $-3x^3 + 9x^2 - 12$

10.) $2x^3 - 4x^2 - 4x - 6$