Constant & Linear Polynomials

Constant polynomials
A constant polynomial is the same thing as a constant function. That is, a constant polynomial is a function of the form
\[ p(x) = c \]
for some number \( c \). For example, \( p(x) = -\frac{5}{3} \) or \( q(x) = -7 \).

The output of a constant polynomial does not depend on the input (notice that there is no \( x \) on the right side of the equation \( p(x) = c \)). Constant polynomials are also called degree 0 polynomials.

The graph of a constant polynomial is a horizontal line. A constant polynomial does not have any roots unless it is the polynomial \( p(x) = 0 \).

Linear polynomials
A linear polynomial is any polynomial defined by an equation of the form
\[ p(x) = ax + b \]
where \( a \) and \( b \) are real numbers and \( a \neq 0 \). For example, \( p(x) = 3x - 7 \) and \( q(x) = -\frac{13}{4}x + \frac{5}{3} \) are linear polynomials. A linear polynomial is the same thing as a degree 1 polynomial.

Roots of linear polynomials
Every linear polynomial has exactly one root. Finding the root is just a matter of basic algebra.

Problem: Find the root of \( p(x) = 3x - 7 \).

Solution: The root of \( p(x) \) is the number \( \alpha \) such that \( p(\alpha) = 0 \). In this problem that means that \( 3\alpha - 7 = 0 \). Hence \( 3\alpha = 7 \), so \( \alpha = \frac{7}{3} \). Thus, \( \frac{7}{3} \) is the root of \( 3x - 7 \).
Slope

The slope of a line is the ratio of the change in the second coordinate to the change in the first coordinate. In different words, if a line contains the two points \((x_1, y_1)\) and \((x_2, y_2)\), then the slope is the change in the \(y\)-coordinate – which equals \(y_2 - y_1\) – divided by the change in the \(x\)-coordinate – which equals \(x_2 - x_1\).

Slope of line containing \((x_1, y_1)\) and \((x_2, y_2)\):

\[
\frac{y_2 - y_1}{x_2 - x_1}
\]

Example: The slope of the line containing the two points \((-1, 4)\) and \((2, 5)\) equals

\[
\frac{5 - 4}{2 - (-1)} = \frac{1}{3}
\]
Graphing linear polynomials

Let \( p(x) = ax \) where \( a \) is a number that does not equal 0. This polynomial is an example of a linear polynomial.

The graph of \( p(x) = ax \) is a straight line that passes through \((0, 0) \in \mathbb{R}^2\) and has slope equal to \( a \). We can check this by graphing it. The point \((0, a0) = (0, 0)\) is in the graph, as are the points \((1, 2a), (2, 3a), \ldots\) and \((-1, -a), (-2, -2a), (-3, -3a), \ldots\)

Because the graph of \( ax + b \) is the graph of \( ax \) shifted up or down by \( b \) – depending on whether \( b \) is positive or negative – the graph of \( ax + b \) is a straight line that passes through \((0, b) \in \mathbb{R}^2\) and has slope equal to \( a \).

**Problem:** Graph \( p(x) = -2x + 4 \).

**Solution:** The graph of \(-2x + 4\) is the graph of \(-2x\) “shifted up” by 4. Draw \(-2x\), which is the line of slope \(-2\) that passes through \((0, 0)\), and then shift it up to the line that passes through \((0, 4)\) and is parallel to \(-2x\).
Another solution: To graph a linear polynomial, find two points in the graph, and then draw the straight line that passes through them.

Since \( p(x) = -2x + 4 \) has 2 as a root, it has an \( x \)-intercept at 2. The \( y \)-intercept is the point in the graph whose first coordinate equals 0, and that’s the point \((0, p(0)) = (0, 4)\). To graph \(-2x + 4\), draw the line passing through the \( x \)- and \( y \)-intercepts.

Behind the name. Degree 1 polynomials are called linear polynomials because their graphs are straight lines.
Exercises

For #1-3, match the numbered constant polynomials with their lettered graphs.

1.) \( p(x) = 3 \)  
2.) \( q(x) = -2 \)  
3.) \( f(x) = \pi \)

A.)  
B.)  
C.)

Find the root for each of the linear polynomials given in #4-9.

4.) \( p(x) = 2x - 3 \)  
5.) \( q(x) = x + 2 \)  
6.) \( r(x) = -\frac{4}{3}x + \frac{6}{7} \)

7.) \( f(x) = 4x - 6 \)  
8.) \( g(x) = \frac{2}{5}x - \frac{8}{5} \)  
9.) \( h(x) = x - 3 \)

For #10-13, find the slope of the line that passes through the two points that are given.

10.) (2, 3) and (3, 5)  
11.) (4, 5) and (−2, 7)  
12.) (−3, 4) and (10, 0)  
13.) (1, −5) and (3, 2)
For #14-16, match the given slope of a line with the lettered lines drawn.

14.) slope $-3$  
15.) slope 2  
16.) slope 0

A.) 

B.) 

C.)

For each of the linear polynomials given in #17-20, find the slope, $x$-intercept, and $y$-intercept of its graph. The slope of the graph of a linear polynomial is its leading coefficient. The $x$-intercept is the root of the linear polynomial. The $y$-intercept is its constant term.

17.) $p(x) = 2x + 1$  
18.) $q(x) = x - 5$  
19.) $f(x) = -3x + 4$  
20.) $g(x) = 4x - 7$
For #21-23, match the given linear polynomial with its lettered graph.

21.) \( p(x) = 2x + 6 \) \hspace{1cm} 22.) \( q(x) = -3x - 2 \) \hspace{1cm} 23.) \( f(x) = x - 4 \)

24.) Claudia owns a coconut collecting company. She has to pay $200 for a coconut collecting license to run her company, and she makes $3 for every coconut she collects. If \( x \) is the number of coconuts she collects, and \( p(x) \) is the number of dollars her company earns, then find an equation for \( p(x) \).

25.) Spencer is payed $400 to collect coconuts no matter how many coconuts he collects. Because he is collecting coconuts for a flat fee, the local government does not require Spencer to purchase a coconut collecting license. If \( q(x) \) is the number of dollars he earns for collecting \( x \) coconuts, what is the equation that defines \( q(x) \)?

26.) If Claudia and Spencer collect the same number of coconuts, then how many coconuts would Claudia have to collect for her company to earn at least as much money as Spencer?