Final Prep: Using inverse functions

Write the following numbers as rational numbers in standard form:

$7^2 = 49$

$3^{-1} = \frac{1}{3}$

$27^{1/3} = \sqrt[3]{27} = 3$

$\left(\frac{4}{9}\right)^{-3/2} = \left(\frac{9}{4}\right)^{3/2} = \left(\sqrt[2]{\frac{9}{4}}\right)^3 = \left(\frac{\sqrt[2]{9}}{\sqrt[2]{4}}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$

$\log_e(e^5) = 5$

$e^{\log_e(7)} = 7$

$\log_2(8) = \log_2(2^3) = 3$

$\log_3\left(\frac{1}{9}\right) = \log_3(3^{-2}) = -2$

$\log_{10}(100{,}000{,}000) = \log_{10}(10^8) = 8$

$\log_4\left(\frac{1}{\sqrt[3]{16}}\right) = \log_4\left(4^{-2/3}\right) = -\frac{2}{3}$

$\log_2(-4)$ ← This expression doesn't make sense. We can only take logarithms of positive numbers, and -4 isn't positive.

(The last problem is a trick question. Why?)

Solve for x

① $3x = 2$ $\quad x = 2/3$

② $\sqrt{2}\,x = 5$ $\quad x = \frac{5}{\sqrt{2}}$

③ $x^3 = 7$ $\quad x = \sqrt[3]{7}$

④ $e^x = 4$ $\quad x = \log_e(4)$

⑤ $\log_e(x) = -5$ $\quad x = e^{-5}$

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⑥ $3x + 5 = 7$

$3x = 2$, $\quad x = 2/3$

⑦ $5\sqrt[3]{x-2} = -2$

$\sqrt[3]{x-2} = -2/5$, $\quad x - 2 = \left(-2/5\right)^3 = \frac{-8}{125}$, $\quad x = \frac{-8}{125} + 2 = \frac{-8}{125} + \frac{250}{125}$

$= \frac{242}{125}$

⑧ $e^{2x-7} = 4$

$2x - 7 = \log_e(4)$, $\quad 2x = \log_e(4) + 7$, $\quad x = \frac{\log_e(4)+7}{2}$

⑨ $\log_e(9-2x) = -5$

$9 - 2x = e^{-5}$, $\quad -2x = e^{-5} - 9$, $\quad x = \frac{e^{-5}-9}{-2} = \frac{9-e^{-5}}{2}$

Complete the rules

$e^x e^y = e^{x+y}$

$\frac{e^x}{e^y} = e^{x-y}$

$(e^x)^y = e^{xy}$

$e^0 = 1$

$\log_e(zw) = \log_e(z) + \log_e(w)$

$\log_e(z/w) = \log_e(z) - \log_e(w)$

$\log_e(z^w) = w\log_e(z)$

$\log_e(1) = 0$

$x^n y^n = (xy)^n$

$\frac{x^n}{y^n} = \left(\frac{x}{y}\right)^n$

$1^n = 1$

$0^n = 0$

$\sqrt[n]{x}\sqrt[n]{y} = \sqrt[n]{xy}$

$\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$

$\sqrt[n]{1} = 1$

$\sqrt[n]{0} = 0$

Solve for x:

⑩ $(x-2)^3 x^3 = -1$

$[(x-2)x]^3 = -1$, $(x-2)x = \sqrt[3]{-1} = -1$, $x^2 - 2x = -1$, $x^2 - 2x + 1 = 0$

$$x = \frac{2 \pm \sqrt{4-4}}{2} = 1$$

⑪ $\frac{\sqrt[3]{x^2+x}}{\sqrt[3]{x}} = -5$

$\sqrt[3]{\frac{x^2+x}{x}} = -5$, $\sqrt[3]{x+1} = -5$, $x+1 = (-5)^3 = -125$, $x = -126$

⑫ $\log_e(x^5) = 5$

$5\log_e(x) = 5, \quad \log_e(x) = 1, \quad x = e$

⑬ $\log_e(x) + \log_e(x-1) = 0$

$\log_e(x(x-1)) = 0, \quad x(x-1) = e^0 = 1, \quad x^2 - x = 1, \quad x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$

so $x = \frac{1+\sqrt{5}}{2}$ or $x = \frac{1-\sqrt{5}}{2}$

Except: $x = \frac{1-\sqrt{5}}{2}$ is not a solution to the given equation. Indeed, $\frac{1-\sqrt{5}}{2} < 0$, so $\log_e\left(\frac{1-\sqrt{5}}{2}\right) + \log_e\left(\frac{1-\sqrt{5}}{2} - 1\right)$ does not make sense (see bottom of page 1). Since it doesn't make sense, it is not equal to 0. Answer: $x = \frac{1+\sqrt{5}}{2}$

⑭ $e^{x^3+x} = 3e^x$

$\frac{e^{x^3+x}}{e^x} = 3, \quad e^{x^3+x-x} = 3, \quad e^{x^3} = 3, \quad x^3 = \log_e(3), \quad x = \sqrt[3]{\log_e(3)}$

⑮ $(e^{x+1})^x = 1$

$e^{(x+1)x} = 1, \quad e^{x^2+x} = 1, \quad x^2+x = \log_e(1) = 0, \quad x(x-1) = 0$

so $x = 0$ or $x = 1$ are the two solutions to the given equation.