Final Prep : Early material

① What is an arithmetic sequence?

A sequence defined by the rule $a_{n+1} = a_n + d$ for some $d \in \mathbb{R}$.

② What is a geometric sequence?

A sequence defined by the rule $a_{n+1} = r a_n$

③ What's the 170^{th} term of $3, 5, 7, 9, \ldots$?

$3 + 169(2) = 3 + 338 = 341$

④ What's the 214^{th} term of $-5, 15, -45, 135, \ldots$?

$(-5)(-3)^{213}$

⑤ For which kinds of sequences do you know how to find the sum of the first k terms?

arithmetic sequences

⑥ For which kinds of sequences have we learned how to find the sum of all the terms?

geometric sequences where $-1 < r < 1$.

⑦ What's the sum of the first 40 terms of $-7, -4, -1, 2, \ldots$?

$$\frac{40}{2}[a_1 + a_{40}] = 20[-7 + (-7 + 39(3))] = 20[-7 - 7 + 117]$$
$$= 20[103] = 2{,}060$$

⑧ What's $\sum_{i=1}^{\infty} \frac{7}{5^i}$?

$$\frac{\frac{7}{5}}{1 - \frac{1}{5}} = \frac{7/5}{4/5} = 7/4$$

⑨ Find $\sum_{i=1}^{3}(1-i^2)$

$(1-1^2)+(1-2^2)+(1-3^2) = 0-3-8 = -11$

⑩ Find $\sum_{i=1}^{30} 5$

$5(30) = 150$

⑪ How many ways are there to order a set of 48 objects?

$48!$

⑫ How many ways can you choose and then order 17 objects from a set of 58 objects?

$\frac{58!}{(58-17)!} = \frac{58!}{41!}$

⑬ How many subsets of a set of 98 objects contain exactly 23 objects?

$\binom{98}{23}$

⑭ What does "options multiply" mean?

To find the number of ways a sequence of decisions can be made, count how many ways each of those decisions can be made, and then multiply each of those numbers.

⑮ Write $\binom{7}{4}$ as a natural number in standard form.

$\frac{7!}{4!3!} = \frac{7\cdot6\cdot5\cdot4!}{4!\,6} = 7\cdot5 = 35$

(16) $f(x)=x^2+2$, $g(x)=3x-1$. Find $fog(x)$ and $gof(x)$.

$fog(x)=f(3x-1)=(3x-1)^2+2$

$gof(x)=g(x^2+2)=3(x^2+2)-1$

(17) $f(x)=2(x-4)^3+1$. Find $f^{-1}(y)$

$y=2(x-4)^3+1$

$y-1=2(x-4)^3$

$\frac{y-1}{2}=(x-4)^3$

$\sqrt[3]{\frac{y-1}{2}}=x-4$

$x=\sqrt[3]{\frac{y-1}{2}}+4$

$f^{-1}(y)=\sqrt[3]{\frac{y-1}{2}}+4$

(18) What are the implied domains of the following functions:

$f(x)=x$ $\mathbb{R}$

$f(x)=x^2$ $\mathbb{R}$

$f(x)=x^3$ $\mathbb{R}$

$f(x)=4$ $\mathbb{R}$

$f(x)=\sqrt[2]{x}$ $[0,\infty)$

$f(x)=\sqrt[3]{x}$ $\mathbb{R}$

$f(x)=1/x$ $\mathbb{R}-\{0\}$

$f(x)=e^x$ $\mathbb{R}$

$f(x)=\log_e(x)$ $(0,\infty)$

(19) What's the implied domain of $f(x) = \frac{27}{3}x^5 - 3x^2 + 27$?

$\mathbb{R}$

(20) What's the implied domain of $r(x) = \frac{3x-7}{x^2-4}$?

$x^2 - 4 \neq 0$

$x^2 \neq 4$

$x \neq 2$ or -2

$\mathbb{R} - \{-2, 2\}$

(21) What's the implied domain of $g(x) = \frac{1}{e^x}$?

$\mathbb{R}$

(22) What's the implied domain of $\sqrt[2]{7-x}$?

$7 - x \geq 0$

$7 \geq x$

$(-\infty, 7]$

(23) What's the implied domain of $5x^2 - \sqrt[3]{2x-3}$?

$\mathbb{R}$

(24) What's the implied domain of $2x - \log_e(3x+4)$?

$3x + 4 > 0$

$3x > -4$

$x > -4/3$

$(-4/3, \infty)$

(25) What's the implied domain of $e^{\sqrt[3]{5x-2}} + 3x^2 - 5$?

$\mathbb{R}$