$p(x) = 9x^3 - 21x^2 + 4x + 4$

① What are the factors of the degree 0 coefficient.

$1, -1, 2, -2, 4, -4$

② Find a root of $p(x)$.

$p(2) = 9(2)^3 - 21(2)^2 + 4(2) + 4$

$= 9 \cdot 8 - 21 \cdot 4 + 8 + 4$

$= 72 - 84 + 8 + 4$

$= 0$

so 2 is a root.

③ Name a linear factor of $p(x)$.

$x - 2$

④ Divide $p(x)$ by the linear factor to find a quadratic factor of $p(x)$.

$$\begin{array}{r|cccc} 2 & 9 & -21 & 4 & 4 \\ & & 18 & -6 & -4 \\ \hline & 9 & -3 & -2 & 0 \end{array}$$

$$\frac{9x^3 - 21x^2 + 4x + 4}{x - 2} = 9x^2 - 3x - 2$$

⑤ How many roots does $9x^2-3x-2$ have?

$(-3)^2-4(9)(-2) = 9+72 = 81 > 0$, so 2 roots.

⑥ What are the roots of $9x^2-3x-2$?

$$\frac{3+\sqrt{81}}{2(9)} = \frac{3+9}{18} = \frac{12}{18} = \frac{2}{3}$$

and $\frac{3-9}{18} = \frac{-6}{18} = -\frac{1}{3}$

⑦ Completely factor $9x^2-3x-2$.

$$9x^2-3x-2 = 9\left(x+\tfrac{1}{3}\right)\left(x-\tfrac{2}{3}\right)$$

⑧ Completely factor $p(x) = 9x^3-21x^2+4x+4$.

$$p(x) = (x-2)(9x^2-3x-2)$$
$$= (x-2)(9)\left(x+\tfrac{1}{3}\right)\left(x-\tfrac{2}{3}\right)$$