Review for second exam

There are no explanations on this list. If anything seems unfamiliar, look it up in the text, or ask about it in class, or ask me or someone else outside of class.

(•) If $f(a)=b$, then $a=f^{-1}(b)$. (If f^{-1} exists.)

(•) If $f(x)$ is written as an equation (for example $f(x)=5x-3$) and you are asked to find $f^{-1}(y)$, then do these three steps:

① Replace $f(x)$ with y

② Solve for x

③ Replace x with $f^{-1}(y)$

(•) If $p(x)$ is a polynomial:

α is a root of $p(x)$ $\longleftrightarrow$ $(x-\alpha)$ is a factor of $p(x)$

(•) Completing the square : $ax^2+bx+c = a\left(x+\dfrac{b}{2a}\right)^2 + c - \dfrac{b^2}{4a}$

(•) Discriminant of ax^2+bx+c is b^2-4ac

(•)

# of roots of a quadratic polynomial	discriminant
2	>0
1	$=0$
0	<0

(•) Quadratic formula : If ax^2+bx+c has 1 or 2 roots, then those roots are

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{and} \quad \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Factoring

(•) A polynomial is <u>completely factored</u> if it is written as a product of its leading coefficient, and a collection of monic quadratic polynomials that don't have roots, and of monic linear polynomials.

(•) Completely factored linears: $ax+b = a\left(x+\frac{b}{a}\right)$

(•) Completely factored quadratics:

If ax^2+bx+c has no roots : $a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$

If ax^2+bx+c has two roots, namely α_1 and α_2 : $a(x-\alpha_1)(x-\alpha_2)$

If ax^2+bx+c has only one root, namely α_1 : $a(x-\alpha_1)(x-\alpha_1)$

(•) Roots of a polynomial are often factors of the constant term/degree 0 coefficient.

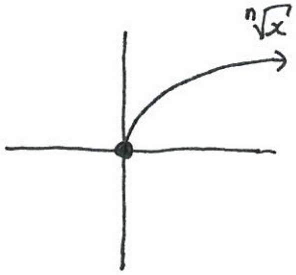
(•) To completely factor a cubic, $p(x)$:

1. Find a root, α, of $p(x)$.
2. Use division to write $p(x)=(x-\alpha)q(x)$ for some quadratic polynomial $q(x)$.
3. Completely factor the quadratic $q(x)$.

Graphs

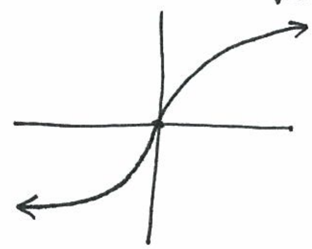
(•) To graph $f^{-1}(x)$, flip the graph of $f(x)$ over the $x=y$ line.

(•) $\sqrt[n]{x}$, n even



Domain is $[0,\infty)$

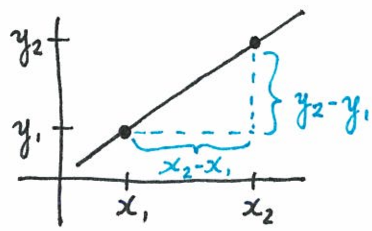
$\sqrt[n]{x}$, n odd $(n \geq 3)$



Domain is $\mathbb{R}$

(•) Roots of a polynomial $p(x)$ are the x-intercepts of its graph.

(•) Slope of a line through (x_1, y_1) and (x_2, y_2) is $\dfrac{y_2 - y_1}{x_2 - x_1}$



(•) To graph $a(x+c)^2+d$, graph ax^2, then shift right or left by c (left if $c>0$, right if $c<0$) and up or down by d (up if $d>0$, down if $d<0$).

Below, $d>0$ and $c>1$.

To graph	Change to graph of $f(x)$
$f(x)+d$	shift up by d
$f(x)-d$	shift down by d.
$cf(x)$	stretch vertically by c
$\frac{1}{c}f(x)$	shrink vertically by $\frac{1}{c}$
$-f(x)$	flip over x-axis

To graph	Change to graph of $f(x)$
$f(x+d)$	shift left by d
$f(x-d)$	shift right by d.
$f(cx)$	shrink horizontally by $\frac{1}{c}$
$f(\frac{1}{c}x)$	stretch horizontally by c.
$f(-x)$	flip over y-axis.