Last Name _____ First Name _____

1.) 36

2.) 2

3.) 89

4.) $2(5)^{88}$

5.) 1800

6.) 30

7.) $\binom{143}{36}$

8.) $\frac{189!}{166!}$

9.) 27!

10.) 20

11.) $27/8$

12.) 5

13.) 1

14.) 0

15.) $-2/9$

16.) 35

17.) $\frac{\log_e(5)-3}{2}$

18.) e^{31}

19.) 3

20.) $g^{-1}(y) = \sqrt[3]{y/8} + 7$

21.) $\mathbb{R}$

22.) $[5/3, \infty)$

23.) $2x + \frac{-x+8}{x^2-1}$

24.) $3(x+1)^2 - 2$

25.) 2

26.) 1

27.) $(x+3)(x+2+\sqrt{3})(x+2-\sqrt{3})$

28.) x and y

29.) $-2/3 < x < 4$

30.) -10

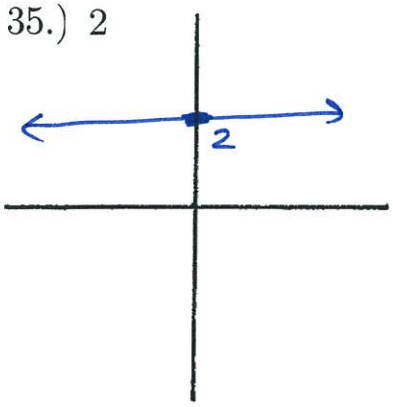
31.) $\begin{pmatrix} 5 & 4 \\ 1 & 1 \end{pmatrix}$

32.) $\begin{pmatrix} 1 & 0 \\ -5/2 & 1/2 \end{pmatrix}$

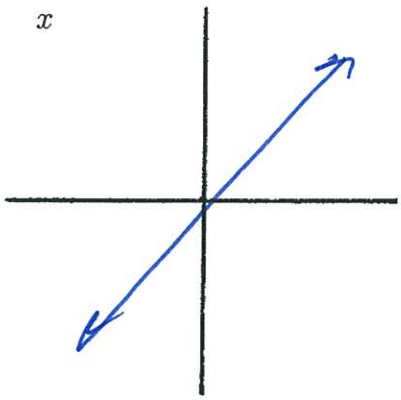
33.) $\begin{pmatrix} 0 & -6 & 2 \\ 0 & -1 & 8 \\ -2 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \\ 4 \end{pmatrix}$

34.) $x = 4,\ y = -7,\ z = -18$

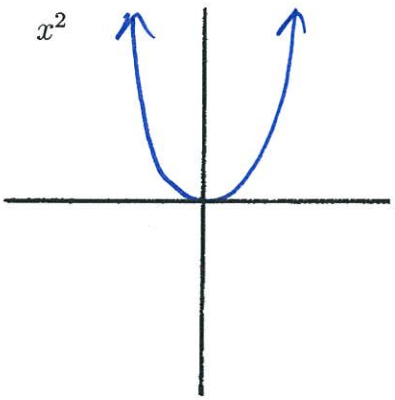
35.) 2



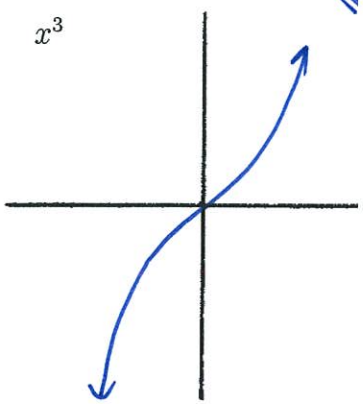
x



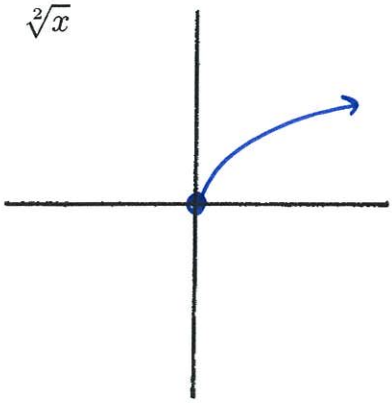
x^2



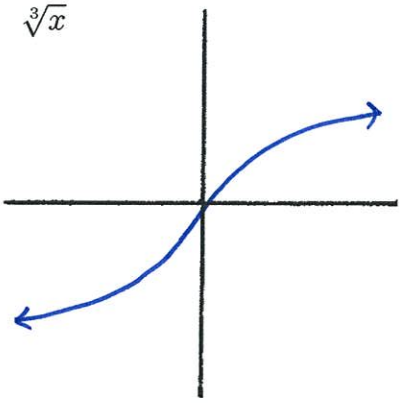
x^3



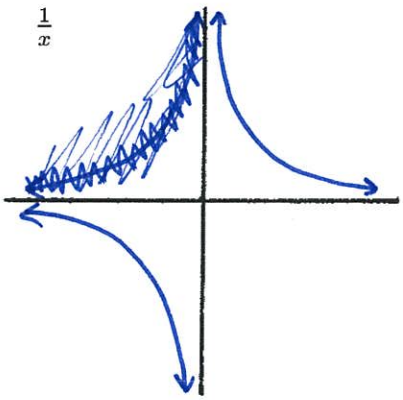
$\sqrt[2]{x}$



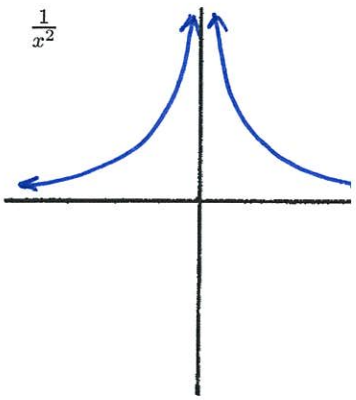
$\sqrt[3]{x}$



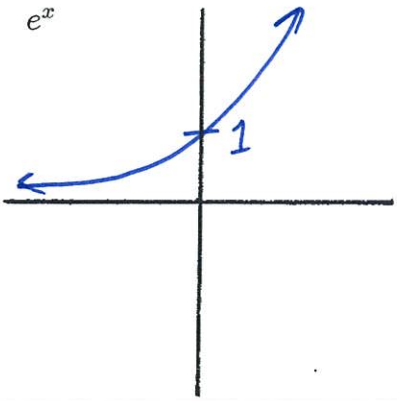
$\frac{1}{x}$



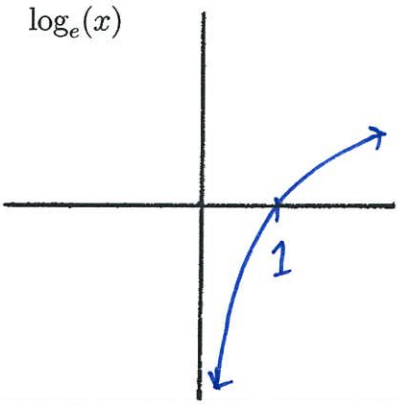
$\frac{1}{x^2}$



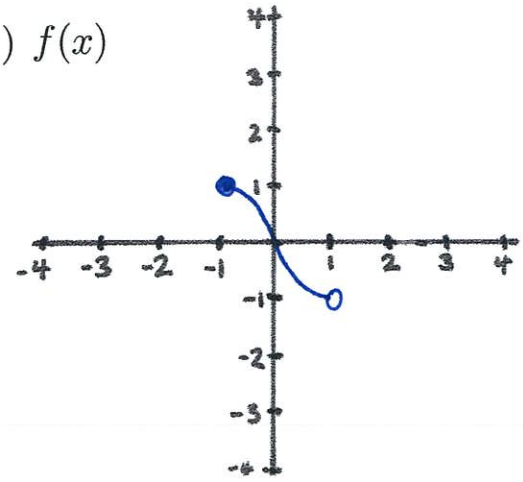
e^x



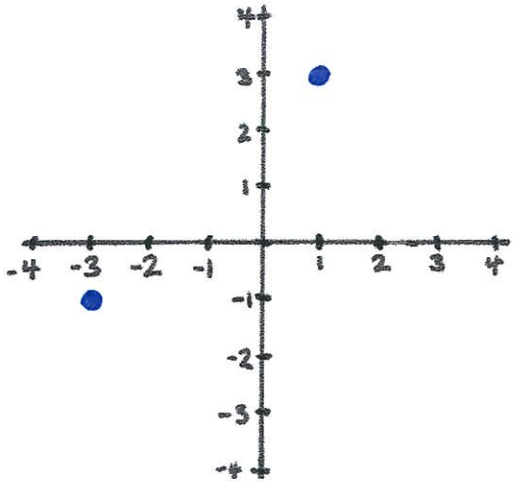
$\log_e(x)$



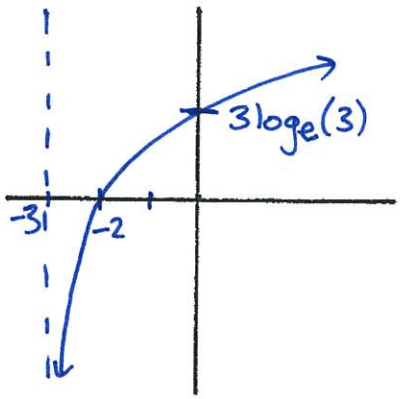
36.) $f(x)$



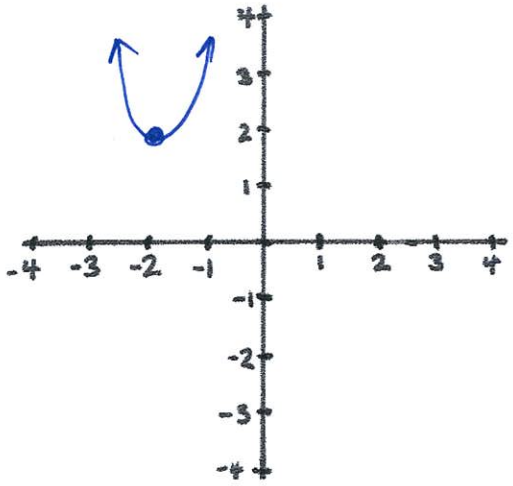
37.) $g(x)$



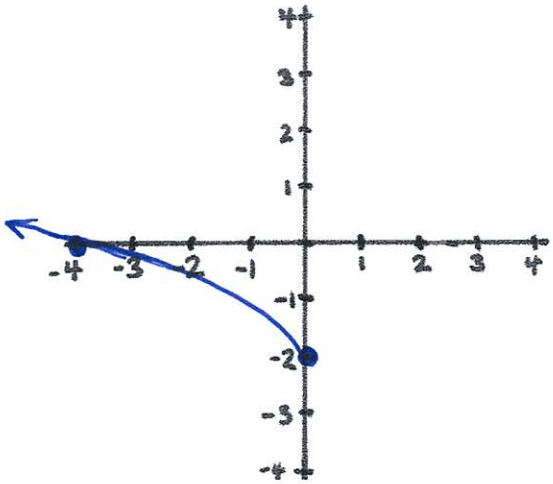
38.) $3\log_e(x+3)$



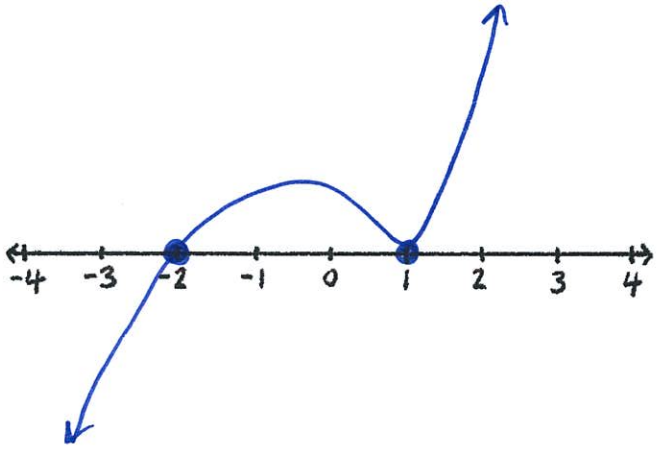
39.) $3(x+2)^2+2$



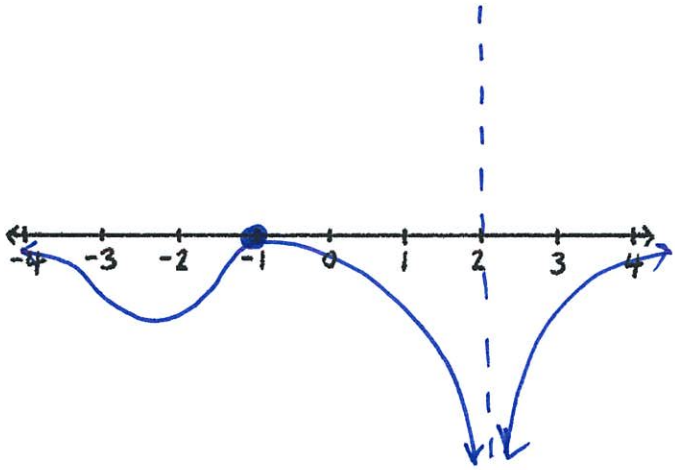
40.) $\sqrt[3]{-x}-2$



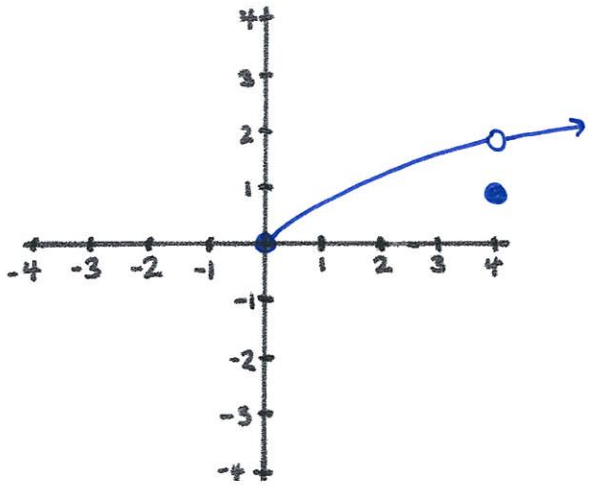
41.) $p(x)$



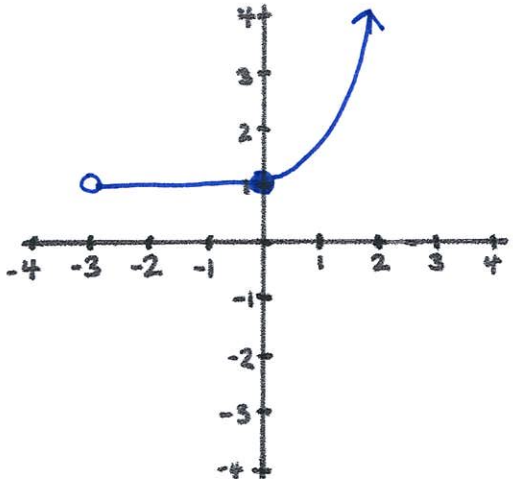
42.) $r(x)$



43.) $h(x)$



44.) $m(x)$



Final Exam

Discrete math

1.) Find $\sum_{i=1}^{3} i^3$

$1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36$

2.) Find $\sum_{i=1}^{\infty} \frac{4}{3^i}$

$\frac{4/3}{1 - 1/3} = \frac{4/3}{2/3} = 2$

3.) What is the 31st term of the sequence $-1, 2, 5, 8, \ldots$?

$-1 + 30(3) = -1 + 90 = 89$

4.) What's the 89th term of $2, 10, 50, 250, \ldots$?

$2(5)^{88}$

5.) What's the sum of the first 30 terms of the sequence $2, 6, 10, 14, \ldots$?

$\frac{30}{2}[2 + 2 + 4(29)] = 15[4(30)] = 15(120)$

$= 1200 + 600$

$= 1800$

6.) You've decided to watch two football games on Sunday, one at 11am and one at 2pm. There are 10 games at 11am and 3 games at 2pm. How many different pairs of games can you watch?

$3(10) = 30$

7.) Suppose a set A contains 143 objects. How many 36 object subsets of A are there?

$\binom{143}{36}$

8.) How many ways are there to choose and order 23 objects from a collection of 189 objects?

$$\frac{189!}{(189-23)!} = \frac{189!}{166!}$$

9.) How many different ways are there to order 27 different objects?

$27!$

10.) Write $\binom{6}{3}$ as an integer in standard form.

$$\frac{6!}{3!3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3!}{6 \cdot 3!} = 5 \cdot 4 = 20$$

****

Algebra

11.) Write $(\frac{4}{9})^{-\frac{3}{2}}$ as a rational number in standard form.

$\left(\frac{9}{4}\right)^{3/2} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$

12.) Write $(25^{-\frac{1}{3}})^2(25^{-\frac{1}{2}})^{-1}\sqrt[3]{25^2}$ as a rational number in standard form.

$25^{-2/3 + 1/2 + 2/3} = 25^{1/2} = \sqrt{25} = 5$

13.) If $a \neq 0$, then what is a^0 as a rational number in standard form? (Notice that this is asking for the y-intercept of the graph of a^x.)

1

14.) If $a > 0$, what is $\log_a(1)$ as a rational number in standard form? (The answer has something to do with the x-intercept of the graph of $\log_a(x)$.)

0

15.) Write $\log_7\left(\sqrt[9]{\frac{1}{49}}\right)$ as a rational number in standard form.

$\log_7\left(7^{-2/9}\right) = -2/9$

16.) Find x where

$$\frac{\sqrt[5]{x^2-3x}}{\sqrt[5]{x}} = 2$$

$\sqrt[5]{x-3} = \sqrt[5]{\frac{x^2-3x}{x}} = 2$

$x-3 = 2^5 = 32 \quad \Rightarrow \quad x = 35$

17.) Find x where $e^{3x} = 5e^{x-3}$.

$e^{3x-x+3} = 5$

$e^{2x+3} = 5$

$2x+3 = \log_e(5)$

$2x = \log_e(5) - 3$

$x = \frac{\log_e(5)-3}{2}$

18.) Find x where $\log_e(\frac{1}{x^2}) = 31 - \log_e(x^3)$.

$\log_e(1/x^2) + \log_e(x^3) = 31$

$\log_e(x^3/x^2) = 31$

$\log_e(x) = 31$

$x = e^{31}$

19.) Find $g \circ f(x)$ if $f(x) = x^2 + 1$ and $g(x) = 3$.

$g(f(x)) = 3$

20.) Find the inverse of $g(x) = 8(x-7)^3$.

$y = 8(x-7)^3$

$\frac{y}{8} = (x-7)^3$

$x - 7 = \sqrt[3]{\frac{y}{8}}$

$x = \sqrt[3]{\frac{y}{8}} + 7$

$g^{-1}(y) = \sqrt[3]{\frac{y}{8}} + 7$

21.) What is the implied domain of $f(x) = 3x - 7 + e^{2x-5}$?

$\mathbb{R}$

22.) What is the implied domain of $g(x) = e^x + 8\sqrt[2]{3x-5}$?

$3x - 5 \geq 0$

$3x \geq 5$

$x \geq 5/3$

23.) Find $\dfrac{2x^3 - 3x + 8}{x^2 - 1}$

$$
\begin{array}{r|l}
x^2 - 1 & 2x \\
\hline
 & 2x^3 - 3x + 8 \\
 & 2x^3 - 2x \\
\hline
 & -x + 8
\end{array}
$$

24.) Complete the square: Write $3x^2 + 6x + 1$ in the form $\alpha(x+\beta)^2 + \gamma$ where $\alpha, \beta, \gamma \in \mathbb{R}$.

$$3\left(x + \frac{6}{2\cdot 3}\right)^2 + 1 - \frac{6^2}{4(3)} = 3(x+1)^2 + 1 - 3$$

$$= 3(x+1)^2 - 2$$

25.) How many roots does $5x^2 - 7x + 2$ have?

$$(-7)^2 - 4(5)(2) = 49 - 40 = 9 > 0$$

so 2 roots.

26.) Find a root of $x^3 + x^2 - 2$.

Factors of -2: $1, -1, 2, -2$

$$1^3 + 1^2 - 2 = 2 - 2 = 0$$

so 1 is a root.

27.) (2 pts.) Completely factor $x^3 + 7x^2 + 13x + 3$. (Hint: -3 is a root.)

-3	1	7	13	3
		-3	-12	-3
	1	4	1	0

$x^3 + 7x^2 + 13x + 3$

$(x+3)$ (x^2+4x+1)

$(x+2+\sqrt{3})$ $(x+2-\sqrt{3})$

x^2+4x+1 has discriminant

$4^2 - 4(1)(1) = 16 - 4 = 12$,

so 2 roots:

$$\frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm 2\sqrt{3}}{2}$$

$$= -2 \pm \sqrt{3}$$

28.) $|x - y|$ is the distance between which two numbers?

x and y

29.) Solve for x if $|5 - 3x| < 7$.

$-7 < 5 - 3x < 7$ $\qquad$ $4 > x > -2/3$

$-12 < -3x < 2$

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Linear algebra

30.) What's the determinant of the matrix below?

$$\begin{pmatrix} 2 & 2 \\ 1 & -4 \end{pmatrix}$$

$2(-4) - 2 \cdot 1 = -8 - 2 = -10$

31.) Find the product

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \cdot 1 + 3 \cdot 1 & 1 \cdot 1 + 3 \cdot 1 \\ 0 \cdot 2 + 1 \cdot 1 & 0 \cdot 1 + 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 1 & 1 \end{pmatrix}$$

32.) What's the inverse of the matrix below?

$$\begin{pmatrix} 1 & 0 \\ 5 & 2 \end{pmatrix}$$

$$\frac{1}{2} \begin{pmatrix} 2 & 0 \\ -5 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -5/2 & 1/2 \end{pmatrix}$$

33.) Write the following system of three linear equations in three variables as a matrix equation

$$\begin{aligned} -6y+2z &= 7 \\ -y+8z &= 9 \\ -2x+\ y+3z &= 4 \end{aligned}$$

$$\begin{pmatrix} 0 & -6 & 2 \\ 0 & -1 & 8 \\ -2 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \\ 4 \end{pmatrix}$$

34.) Solve for x, y, and z if

$$\begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ -4 & 5 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ -4 & 5 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} (-1)(-4) \\ 2(-4)+(1)(1) \\ (-4)(5)+(2)(1) \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \\ -18 \end{pmatrix}$$

****

Graphs

35.) Graph the following functions: 2, x, x^2, x^3, $\sqrt[2]{x}$, $\sqrt[3]{x}$, $\frac{1}{x}$, $\frac{1}{x^2}$, e^x, $\log_e(x)$.

36.) Graph $f : [-1, 1) \to \mathbb{R}$ where $f(x) = -x^3$.

37.) Graph $g : \{-3, 1\} \to \mathbb{R}$ where $g(x) = x + 2$.

$g(-3) = -3+2 = -1$

$g(1) = 1+2 = 3$

38.) Graph $3\log_e(x+3)$ and label its x- and y-intercepts (if there are any).

vert. stretch; left 3

y-int: $3\log_e(0+3) = 3\log_e(3)$

x-int: $0 = 3\log_e(x+3)$

$\Rightarrow 0 = \log_e(x+3)$

$\Rightarrow 1 = x+3 \Rightarrow x = -2$

39.) Graph $3(x+2)^2 + 2$ and label its vertex.

parabola opens up; left 2; up 2.

40.) Graph $\sqrt[2]{-x} - 2$ and label its x- and y-intercepts (if there are any).

flip over y-axis; down 2

y-int: $\sqrt{-0} - 2 = -2$

x-int: $0 = \sqrt{-x} - 2$

$\Rightarrow 2 = \sqrt{-x}$

$\Rightarrow -x = 4$

$\Rightarrow x = -4$

41.) Graph $p(x)$. (Label all x-intercepts.)

$$p(x) = 3(x+2)(x-1)(x-1)$$

x-int: -2, 1

in between: $p(0) = 3(2)(-1)(-1) > 0$.

left & right: $3x^3$

42.) Graph $r(x)$ (Label all x-intercepts and all vertical asymptotes.)

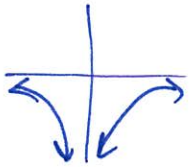
$$r(x) = \frac{-2(x+1)(x+1)}{3(x-2)(x-2)(x^2+1)}$$

vert. asy: 2

x-int: -1

in between: $r(0) = \dfrac{-2(1)(1)}{3(-2)(-2)(1)} < 0$

left & right: $\dfrac{-2x^2}{3x^4} = -\frac{2}{3}\left(\frac{1}{x^2}\right)$



43.) Graph

$$h(x) = \begin{cases} \sqrt[2]{x} & \text{if } x \neq 4; \\ 1 & \text{if } x = 4. \end{cases}$$

44.) Graph

$$m(x) = \begin{cases} 1 & \text{if } x \in (-3, 0); \\ 2^x & \text{if } x \in [0, \infty). \end{cases}$$