Last Name_____ First Name._____

1.) 36

2.) 1

3.) 88

4.) $2(5)^{22}$

5.) 1800

6.) $\frac{126!}{85!}$

7.) $\binom{158}{27}$

8.) 30

9.) 38!

10.) 21

11.) $\frac{27}{8}$

12.) 6

13.) 1

14.) 0

15.) $-\frac{2}{5}$

16.) 34

17.) $\frac{\log_e(7)-2}{2}$

18.) e^{27}

19.) $3x^2+3$ ($3(x^2+1)$ is fine too)

20.) $g^{-1}(y) = \log_e\left(\frac{y}{8}\right)+7$

21.) $\mathbb{R}$

22.) $[7/2, \infty)$

23.) $2x+\frac{-x+1}{x^2-1}$

24.) $3(x+1)^2-2$

25.) 2

26.) -1

27.) $(x-3)(x+2-\sqrt{3})(x+2+\sqrt{3})$

28.) x and y

29.) $-3/2 < x < 9/2$

30.) 6

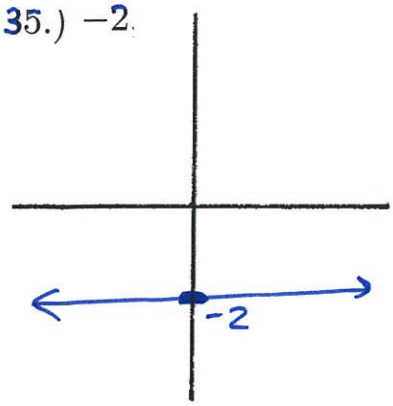
31.) $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$

32.) $\begin{pmatrix} 1/2 & -3/2 \\ 0 & 1 \end{pmatrix}$

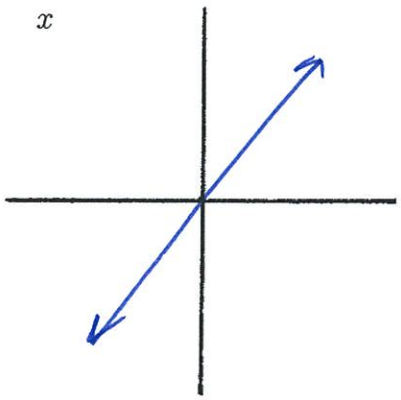
33.) $\begin{pmatrix} 0 & -6 & 3 \\ 0 & -1 & 2 \\ -2 & 1 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 9 \end{pmatrix}$

34.) $x=-1,\ y=-2,\ z=-3$

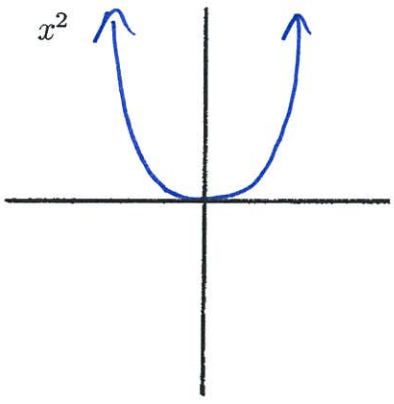
35.) -2



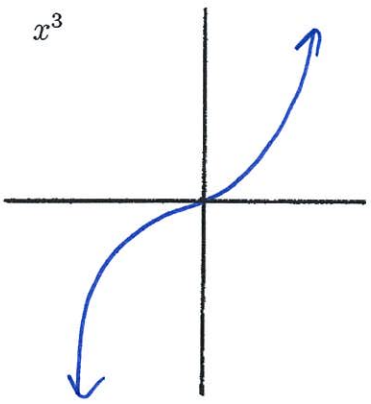
x



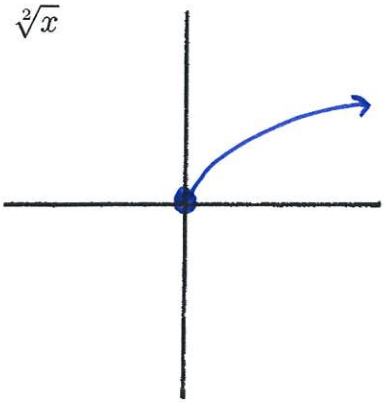
x^2



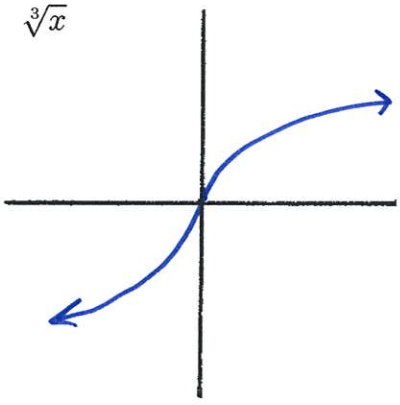
x^3



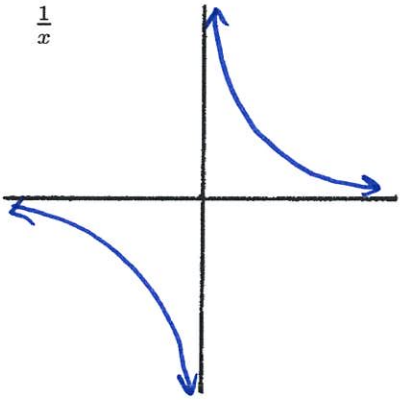
$\sqrt[2]{x}$



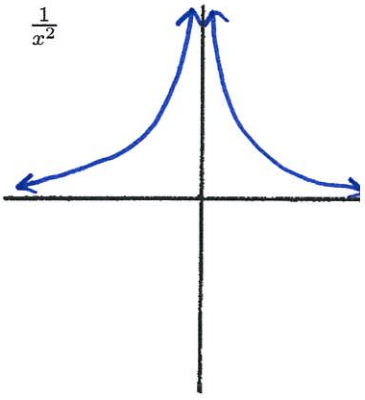
$\sqrt[3]{x}$



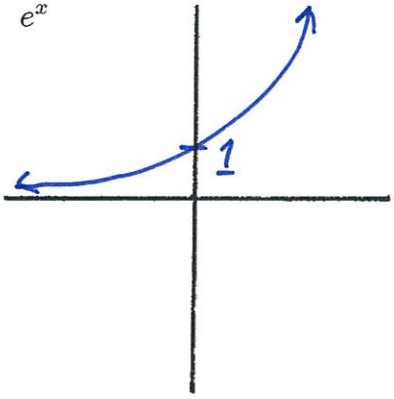
$\frac{1}{x}$



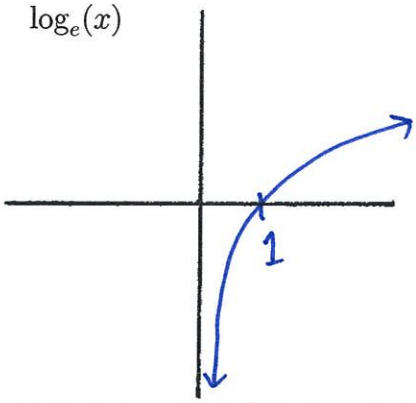
$\frac{1}{x^2}$



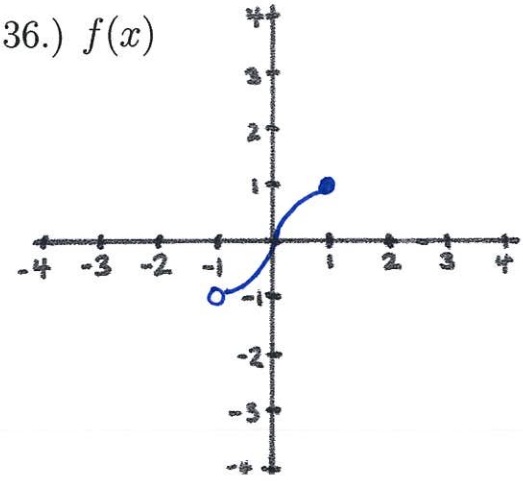
e^x



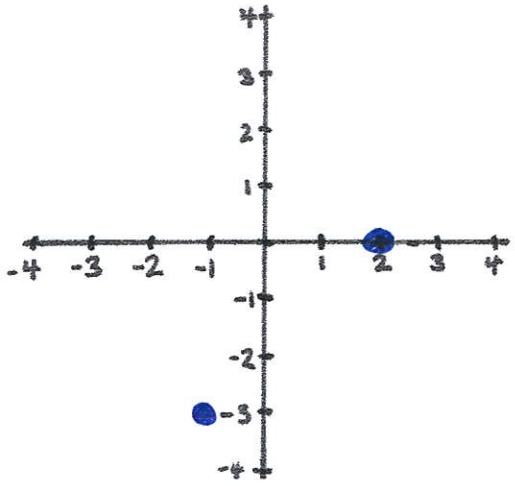
$\log_e(x)$



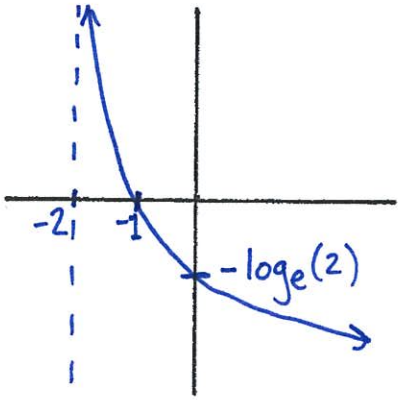
36.) $f(x)$



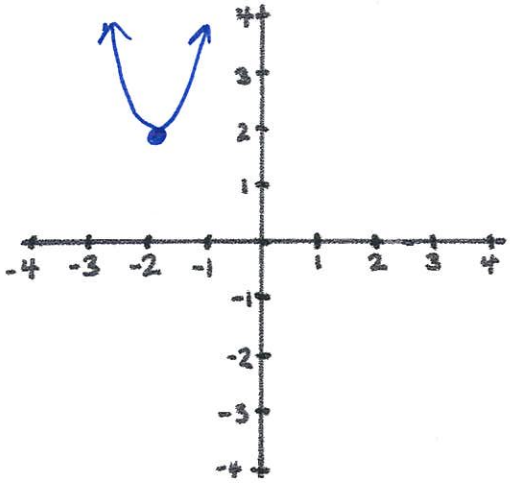
37.) $g(x)$



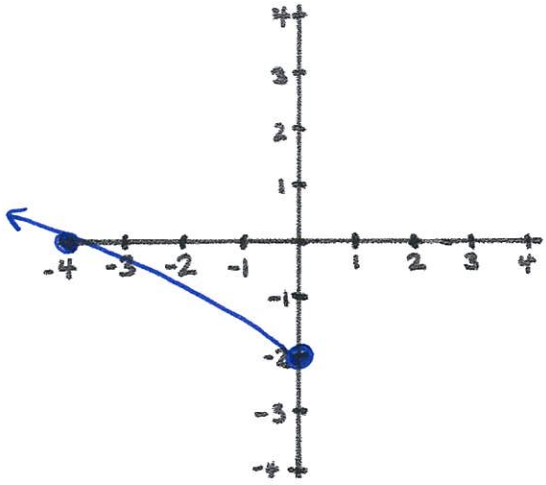
38.) $-\log_e(x+2)$



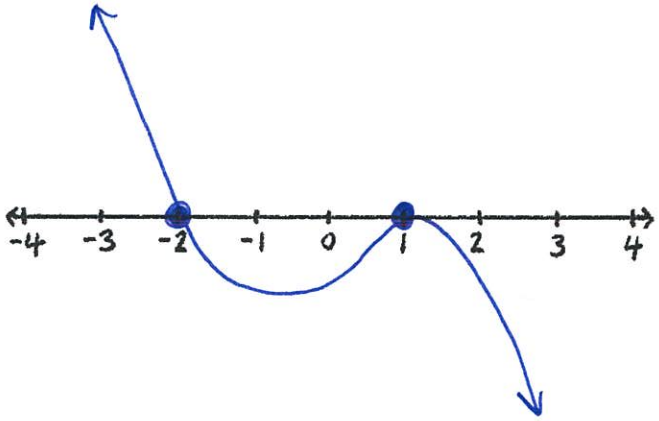
39.) $3(x+2)^2+2$



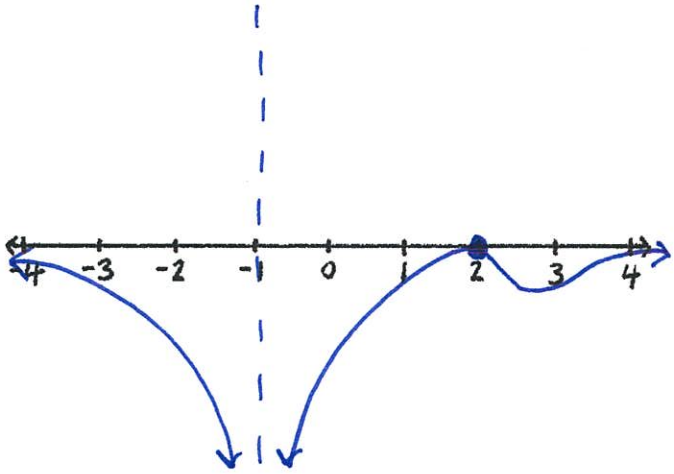
40.) $\sqrt[2]{-x}-2$



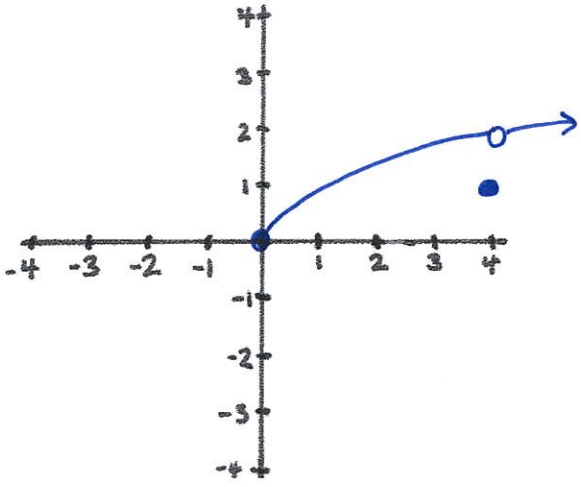
41.) $p(x)$



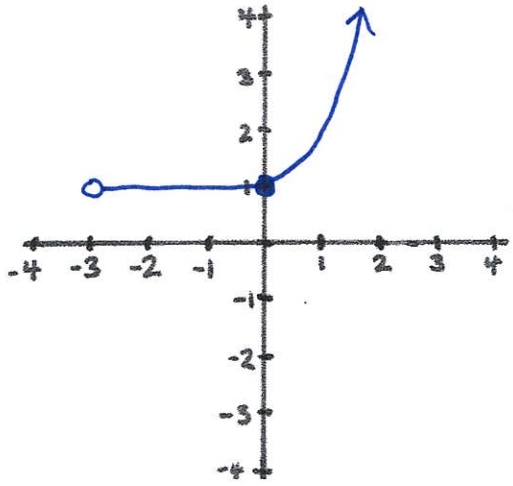
42.) $r(x)$



43.) $h(x)$



44.) $m(x)$



Final Exam

Discrete math

1.) Find $\sum_{i=1}^{3} i^3$

$1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36$

2.) Find $\sum_{i=1}^{\infty} \frac{3}{4^i}$

$\frac{3/4}{1 - 1/4} = \frac{3/4}{3/4} = 1$

3.) What is the 31st term of the sequence $-2, 1, 4, 7, \ldots$?

$-2 + (30)(3) =$

$= -2 + 90 = 88$

4.) What's the 23rd term of $2, 10, 50, 250, \ldots$?

$2(5)^{22}$

5.) What's the sum of the first 30 terms of the sequence $2, 6, 10, 14, \ldots$?

$\frac{30}{2}[2 + 2 + (29)4] = 15(30)(4)$

$= 15(120)$

$= 1{,}800$

$\begin{array}{r} 120 \\ 15 \\ \hline 600 \\ 1200 \\ \hline 1800 \end{array}$

6.) How many ways are there to choose and order 41 objects from a collection of 126 objects?

$$\frac{126!}{(126-41)!} = \frac{126!}{85!}$$

7.) Suppose a set A contains 158 objects. How many 27 object subsets of A are there?

$$\binom{158}{27}$$

8.) You've decided to watch two football games on Sunday, one at 11am and one at 2pm. There are 10 games at 11am and 3 games at 2pm. How many different pairs of games can you watch?

$$(10)(3) = 30$$

9.) How many different ways are there to order 38 different objects?

$$38!$$

10.) Write $\binom{7}{5}$ as an integer in standard form.

$$\frac{7!}{5!\,2!} = \frac{7 \cdot 6 \cdot 5!}{5!\,2} = 7 \cdot 3 = 21$$

****

Algebra

11.) Write $(\frac{4}{9})^{-\frac{3}{2}}$ as a rational number in standard form.

$$\left(\frac{9}{4}\right)^{3/2} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

12.) Write $(36^{-\frac{1}{3}})^2(36^{-\frac{1}{2}})^{-1}\sqrt[3]{36^2}$ as a rational number in standard form.

$$36^{-2/3 + 1/2 + 2/3} = 36^{1/2} = \sqrt{36} = 6$$

13.) If $a \neq 0$, then what is a^0 as a rational number in standard form? (Notice that this is asking for the y-intercept of the graph of a^x.)

1

14.) If $a > 0$, what is $\log_a(1)$ as a rational number in standard form? (The answer has something to do with the x-intercept of the graph of $\log_a(x)$.)

0

15.) Write $\log_9\left(\sqrt[5]{\frac{1}{81}}\right)$ as a rational number in standard form.

$$\log_9\left(9^{-2/5}\right) = -2/5$$

16.) Find x where

$$\frac{\sqrt[5]{x^2-2x}}{\sqrt[5]{x}} = 2$$

$\sqrt[5]{\frac{x^2-2x}{x}} = 2$ | $x-2 = 2^5 = 32$

$\sqrt[5]{x-2} = 2$ | $x = 34$

17.) Find x where $e^{3x} = 7e^{x-2}$.

$e^{3x-x+2} = 7$ | $2x+2 = \log_e(7)$

$e^{2x+2} = 7$ | $2x = \log_e(7) - 2$

$x = \frac{\log_e(7)-2}{2}$

18.) Find x where $\log_e(\frac{1}{x^2}) = 27 - \log_e(x^3)$.

$\log_e(1/x^2) + \log_e(x^3) = 27$

$\log_e(x^3/x^2) = 27$

$\log_e(x) = 27$ $\quad x = e^{27}$

19.) Find $g \circ f(x)$ if $f(x) = x^2 + 1$ and $g(x) = 3x$.

$g(x^2+1) = 3x^2 + 3$

20.) Find the inverse of $g(x) = 8e^{x-7}$.

$y = 8e^{x-7}$

$\frac{y}{8} = e^{x-7}$

$x - 7 = \log_e(\frac{y}{8})$ | $x = \log_e(\frac{y}{8}) + 7$

$g^{-1}(y) = \log_e(\frac{y}{8}) + 7$

21.) What is the implied domain of $f(x) = 3x - 7 + e^{2x-5}$?

$\mathbb{R}$

22.) What is the implied domain of $g(x) = e^x + 8\sqrt[2]{2x-7}$?

$2x - 7 \geq 0$

$2x \geq 7$

$x \geq 7/2$

23.) Find $\dfrac{2x^3 - 3x + 1}{x^2 - 1}$

$$\begin{array}{r|l} & 2x \\ \hline x^2-1 & 2x^3 - 3x + 1 \\ & 2x^3 - 2x \\ \hline & -x + 1 \end{array}$$

24.) Complete the square: Write $3x^2 + 6x + 1$ in the form $\alpha(x + \beta)^2 + \gamma$ where $\alpha, \beta, \gamma \in \mathbb{R}$.

$$3\left(x + \frac{6}{2\cdot 3}\right)^2 + 1 - \frac{6^2}{4(3)} = 3(x+1)^2 - 2$$

25.) How many roots does $5x^2 - 7x + 2$ have?

$$(-7)^2 - 4(5)(2) = 49 - 40 = 9 > 0$$

so 2 roots

26.) Find a root of $x^3 + 3x^2 + 4x + 2$.

Factors of -2 : $1, -1, 2, -2$

$$(-1)^3 + 3(-1)^2 + 4(-1) + 2 = -1 + 3 - 4 + 2 = 0$$

so -1 is a root.

27.) (2 pts.) Completely factor $x^3 + x^2 - 11x - 3$. (Hint: 3 is a root.)

3	1	1	-11	-3
		3	12	3
	1	4	1	0

$x^3 + x^2 - 11x - 3$

$(x-3)$ (x^2+4x+1)

$(x+2-\sqrt{3})$ $(x+2+\sqrt{3})$

discriminant of x^2+4x+1:

$4^2 - 4(1)(1) = 16 - 4 = 12$

so 2 roots:

$$\frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm 2\sqrt{3}}{2}$$

$$= -2 \pm \sqrt{3}$$

28.) $|x - y|$ is the distance between which two numbers?

x and y

29.) Solve for x if $|3 - 2x| < 6$.

$-6 < 3 - 2x < 6$

$-9 < -2x < 3$

$\frac{9}{2} > x > -\frac{3}{2}$

****

Linear algebra

30.) What's the determinant of the matrix below?

$$\begin{pmatrix} 2 & 2 \\ 1 & 4 \end{pmatrix}$$

$2 \cdot 4 - 2 \cdot 1 = 8 - 2 = 6$

31.) Find the product

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \cdot 1 + 1 \cdot 1 & 1 \cdot 1 + 1 \cdot 1 \\ 2 \cdot 0 + 1 \cdot 1 & 0 \cdot 1 + 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$$

32.) What's the inverse of the matrix below?

$$\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$$

$$\frac{1}{2} \begin{pmatrix} 1 & -3 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{3}{2} \\ 0 & 1 \end{pmatrix}$$

33.) Write the following system of three linear equations in three variables as a matrix equation

$$\begin{aligned} -6y+3z &= 4 \\ -y+2z &= 7 \\ -2x+\ y+8z &= 9 \end{aligned}$$

$$\begin{pmatrix} 0 & -6 & 3 \\ 0 & -1 & 2 \\ -2 & 1 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 9 \end{pmatrix}$$

34.) Solve for x, y, and z if

$$\begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -4 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ -4 & 5 & 2 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ -4 & 5 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 2\cdot 1+1(-4) \\ 1\cdot 5+2(-4) \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$$

****

Graphs

35.) Graph the following functions: -2, x, x^2, x^3, $\sqrt[2]{x}$, $\sqrt[3]{x}$, $\frac{1}{x}$, $\frac{1}{x^2}$, e^x, $\log_e(x)$.

36.) Graph $f : (-1, 1] \to \mathbb{R}$ where $f(x) = x^3$.

37.) Graph $g : \{-1, 2\} \to \mathbb{R}$ where $g(x) = x - 2$.

$g(-1) = -1 - 2 = -3$

$g(2) = 2 - 2 = 0$

38.) Graph $-\log_e(x+2)$ and label its x- and y-intercepts (if there are any).

flip over x-axis. left 2

y-int: $-\log_e(0+2) = -\log_e(2)$

x-int: $0 = -\log_e(x+2)$

$\Rightarrow 0 = \log_e(x+2)$

$\Rightarrow 1 = x+2$

$\Rightarrow x = -1$

39.) Graph $3(x+2)^2 + 2$ and label its vertex.

parabola opens up. left 2 up 2.

40.) Graph $\sqrt[2]{-x} - 2$ and label its x- and y-intercepts (if there are any).

flip over y-axis down 2

y-int: $\sqrt{-0} - 2 = -2$

x-int: $0 = \sqrt{-x} - 2$

$\Rightarrow 2 = \sqrt{-x}$

$\Rightarrow -x = 4$

$\Rightarrow x = -4$

41.) Graph $p(x)$. (Label all x-intercepts.)

$$p(x) = -3(x+2)(x-1)(x-1)$$

x-int: -2, 1

in between: $p(0) = -3(2)(-1)(-1) < 0$

left & right: $-3x^3$

42.) Graph $r(x)$ (Label all x-intercepts and all vertical asymptotes.)

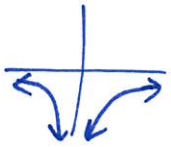
$$r(x) = \frac{-2(x-2)(x-2)}{3(x+1)(x+1)(x^2+1)}$$

vert asy: -1

x-int: 2

in between: $r(0) = \frac{(-2)(-2)(-2)}{(3)(1)(1)(1)} < 0$

left and right: $\frac{-2x^2}{3x^4} = -\frac{2}{3}\frac{1}{x^2}$



43.) Graph

$$h(x) = \begin{cases} \sqrt[2]{x} & \text{if } x \neq 4; \\ 1 & \text{if } x = 4. \end{cases}$$

44.) Graph

$$m(x) = \begin{cases} 1 & \text{if } x \in (-3, 0); \\ 2^x & \text{if } x \in [0, \infty). \end{cases}$$