Last Name:_____ First Name:_____

1.) True

2.) False

3.) True

4.) True

5.) False

6.) True

7.) True

8.) False

9.) -7

10.) 3

11.) $f^{-1}(y) = \left(\frac{y-2}{3}\right)^3 + 1$

12.) $[2/7, \infty)$

13.) 0

14.) $2x^2 - 3x + 4 + \frac{x+3}{3x^2+2}$

15.) $2x^2 - 11x + 22 + \frac{-41}{x+2}$

16.) 3

17.) $4(x+\frac{1}{2})^2 - 4$

18.) 0

19.) $1+\sqrt{11}$ and $1-\sqrt{11}$

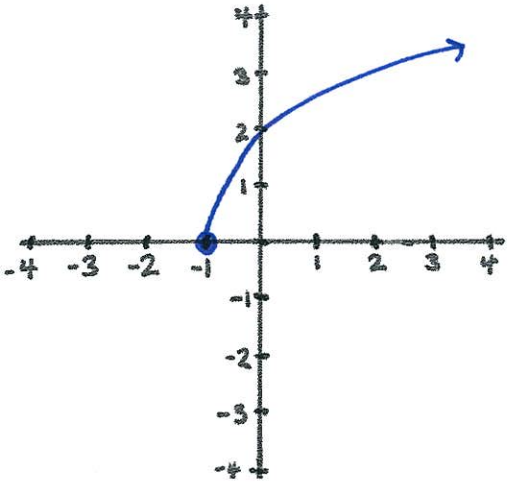
20.) -2

21.) $2(x-2)(x^2+x+2)$

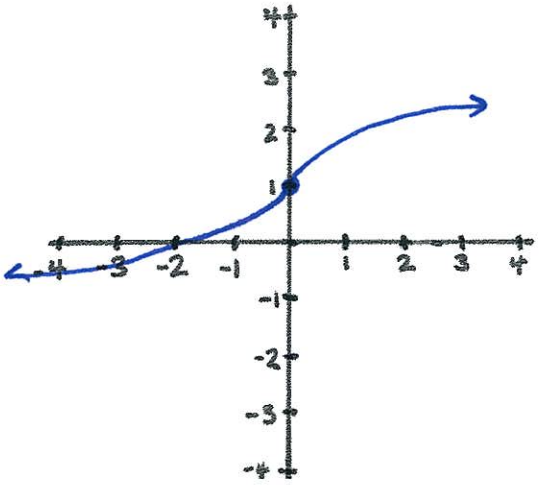
22.) $2(x+2)(x+3)(x-\frac{1}{2})$

23.) $x+2$ $x-3$

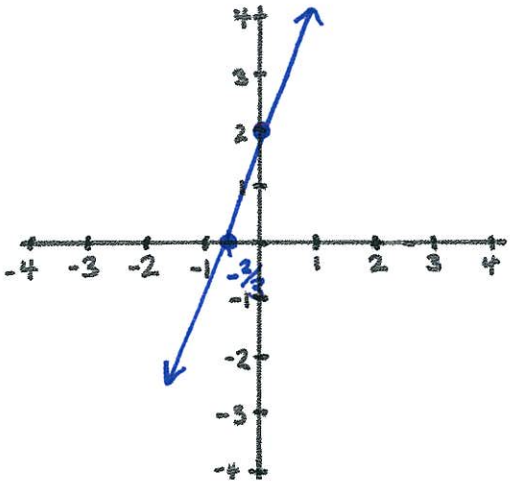
24.) $2\sqrt{x+1}$



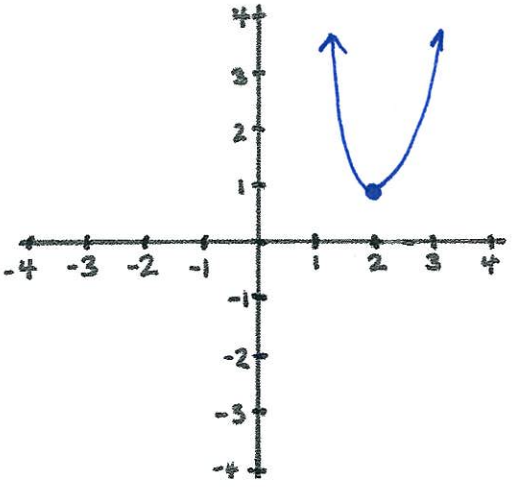
25.) $\sqrt[3]{\frac{x}{2}}+1$



26.) $3x+2$



27.) $3(x-2)^2+1$



Second Exam

True/False

For #1-8 write the entire word "True" or the entire word "False".

1.) $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

2.) $(x+y)^n = x^n + y^n$

3.) $\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$

4.) $(xy)^n = x^n y^n$

5.) $\sqrt[n]{x+y} = \sqrt[n]{x} + \sqrt[n]{y}$

6.) $(\frac{x}{y})^n = \frac{x^n}{y^n}$

7.) $a(b+c) = ab + ac$

8.) $7x^3 + 12x^2 + 5x - 4$ has 4 roots.

****

Algebra

9.) Find x where $(x+4)^3 + 7 = -20$.

$(x+4)^3 = -27$

$x+4 = \sqrt[3]{-27} = -3$

$x = -7$

10.) If $g(x)$ is an invertible function, and $g(3) = 5$, then what is $g^{-1}(5)$?

11.) Find the inverse of $f(x) = 3\sqrt[3]{x-1} + 2$. (You can check your answer by seeing if $f^{-1} \circ f(x) = x$.)

$y = 3\sqrt[3]{x-1} + 2$

$y - 2 = 3\sqrt[3]{x-1}$

$\frac{y-2}{3} = \sqrt[3]{x-1}$

$\left(\frac{y-2}{3}\right)^3 = x - 1$

$x = \left(\frac{y-2}{3}\right)^3 + 1$

$f^{-1}(y) = \left(\frac{y-2}{3}\right)^3 + 1$

12.) What is the implied domain of $g(x) = -\sqrt[2]{7x-2} + 18x^2 - 3$? (Write your answer as an interval.)

$7x - 2 \geq 0$

$7x \geq 2$

$x \geq 2/7$

$[2/7, \infty)$

13.) Suppose that $a \neq 0$ and that $b^2 - 4ac \geq 0$. Write the following number as an integer in standard form:

$$a\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)^2 + b\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right) + c$$

0

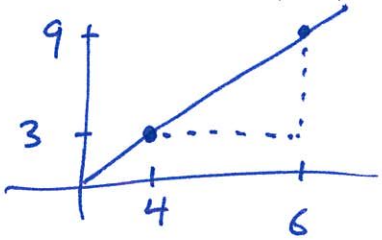
14.) Find $\dfrac{6x^4 - 9x^3 + 16x^2 - 5x + 11}{3x^2 + 2}$

$$
\begin{array}{r}
2x^2 - 3x + 4 \\
3x^2+2 \overline{\big| 6x^4 - 9x^3 + 16x^2 - 5x + 11} \\
\underline{6x^4 \qquad + 4x^2} \\
-9x^3 + 12x^2 - 5x + 11 \\
\underline{-3x^3 \qquad - 6x} \\
12x^2 + x + 11 \\
\underline{12x^2 \qquad + 8} \\
x + 3
\end{array}
$$

15.) Find $\dfrac{2x^3 - 7x^2 + 3}{x + 2}$

$$
\begin{array}{r|cccc}
-2 & 2 & -7 & 0 & 3 \\
 & & -4 & 22 & -44 \\
\hline
 & 2 & -11 & 22 & -41
\end{array}
$$

16.) What is the slope of the straight line in $\mathbb{R}^2$ that passes through the points $(4, 3)$ and $(6, 9)$?



$$\frac{9-3}{6-4} = \frac{6}{2} = 3$$

17.) Complete the square: Write $4x^2 + 4x - 3$ in the form $\alpha(x+\beta)^2 + \gamma$ where $\alpha, \beta, \gamma \in \mathbb{R}$.

$$4\left(x + \frac{4}{2\cdot 4}\right)^2 - 3 - \frac{4^2}{4(4)} = 4\left(x + \frac{1}{2}\right)^2 - 4$$

18.) How many roots does $3x^2 - 2x + 10$ have?

$$(-2)^2 - 4(3)(10) = 4 - 120 < 0$$

so 0 roots

19.) Find the roots of $x^2 - 2x - 10$

$$(-2)^2 - 4(1)(-10) = 4 + 40 = 44$$

$$\frac{2 \pm \sqrt{44}}{2(1)} = \frac{2 \pm 2\sqrt{11}}{2} = 1 \pm \sqrt{11}$$

20.) Find a root of $3x^3 + 8x^2 + 5x + 2$

Factors of 2 : 1, 2, -2, -1.

$$3(-2)^3 + 8(-2)^2 + 5(-2) + 2 = 3(-8) + 8(4) + 5(-2) + 2$$

$$= -24 + 32 - 10 + 2$$

$$= 0$$

so -2 is a root.

21.) (2 points) Completely factor $2x^3 - 2x^2 - 8$. (Hint: 2 is a root.) (Your answer should be a product of a constant and maybe some linear and quadratic polynomials that have leading coefficients equal to 1, and such that any of the quadratics in the product have no roots.)

$$\begin{array}{c|cccc} 2 & 2 & -2 & 0 & -8 \\ & & 4 & 4 & 8 \\ \hline & 2 & 2 & 4 & 0 \end{array}$$

Discriminant of $2x^2+2x+4$ is $2^2-4(2)(4) = 4-32<0$, so factor by factoring out the leading coefficient.

$$2x^3-2x^2-8$$

$$(x-2)\quad(2x^2+2x+4)$$

$$2\quad(x^2+x+2)$$

22.) (2 points) Completely factor $2x^3 + 9x^2 + 7x - 6$. (Hint: -2 is a root.) (Your answer should have the same form as described in #21.)

$$\begin{array}{c|cccc} -2 & 2 & 9 & 7 & -6 \\ & & -4 & -10 & 6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

Discriminant of $2x^2+5x-3$ is $5^2-4(2)(-3) = 25+24 = 49$

2 Roots: $\frac{-5+\sqrt{49}}{2(2)} = \frac{-5+7}{4} = \frac{2}{4} = \frac{1}{2}$

and $\frac{-5-7}{4} = \frac{-12}{4} = -3$

$$2x^3+9x^2+7x-6$$

$$(x+2)\quad(2x^2+5x-3)$$

$$2\quad(x+3)\quad(x-\tfrac{1}{2})$$

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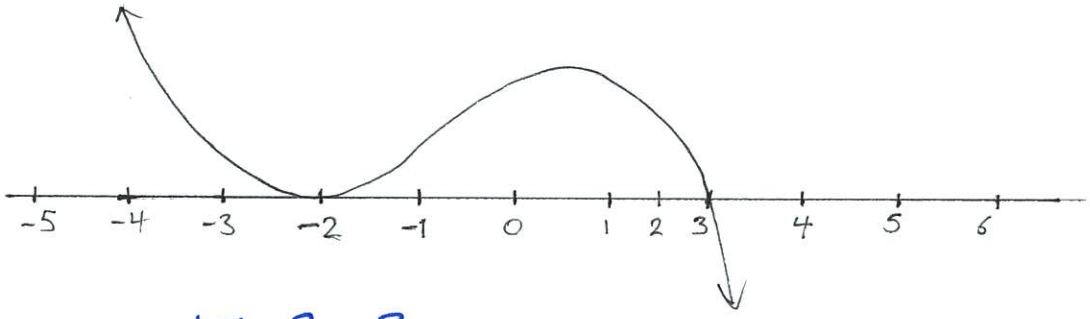
Graphs

23.) List all of the monic linear factors of $p(x)$ that you know of from the graph below.



roots: -2 , 3

factors: $x+2$, $x-3$

24.) Graph $2\sqrt{x+1}$ and label its x- and y-intercepts.

stretch vert. by 2 ; shift left 1.

x-int: $2\sqrt{x+1}=0 \Rightarrow x=-1$

y-int: $2\sqrt{0+1}=2$

25.) Graph $\sqrt[3]{\frac{x}{2}}+1$ and label its x- and y-intercepts.

stretch hor. by 2 ; shift up 1.

x-int: $\sqrt[3]{\frac{x}{2}}+1=0 \Rightarrow x=-2$

y-int: $\sqrt[3]{\frac{0}{2}}+1=1$

26.) Graph $3x+2$ and label its x- and y-intercepts.

x-int: $3x+2=0 \Rightarrow x=-\frac{2}{3}$

y-int: $3\cdot 0+2=2$

27.) Graph $3(x-2)^2+1$ and label its vertex. (You don't have to be accurate with the x- and y-intercepts.)

parabola opens up. ; shift right 2 ; shift up 1.