Last Name: _____ First Name: _____

1.) 26

2.) $2/5$

3.) 247

4.) $(-3)(-2)^{56}$

5.) 3,720

6.) $\binom{243}{92}$

7.) $\frac{304!}{255!}$

8.) $93!$

9.) 66

10.) 84

11.) $4/9$

12.) 49

13.) 1

14.) 0

15.) $-4/5$

16.) $-3+\sqrt{13}$, $-3-\sqrt{13}$

17.) 3

18.) $\sqrt[7]{e^3}$

19.) $(x+2)^2$

20.) $g^{-1}(y) = e^{y/7} - 3$

21.) $(-\infty, 3/7)$

22.) $\mathbb{R}$

23.) $x-3+\frac{-x+2}{x^2-4}$

24.) $-2(x+1)^2-3$

25.) no roots

26.) -3

27.) $(-2)(x-2)(x^2+2x+3)$

28.) x and y

29.) $-2/3 < x < 2$

30.) -7

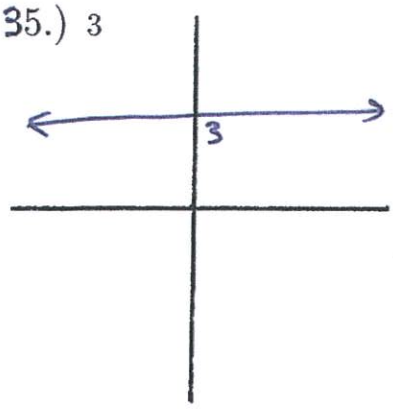
31.) $\begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$

32.) $\begin{pmatrix} -3/5 & 4/5 \\ 2/5 & -1/5 \end{pmatrix}$

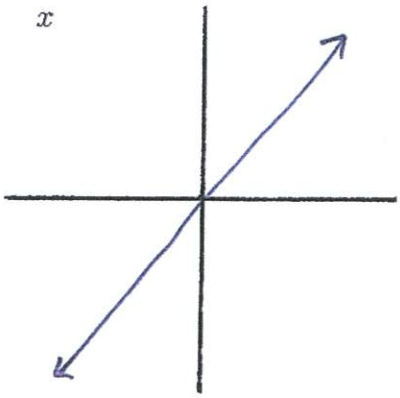
33.) $\begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

34.) $x = -4, y = 7, z = 20$

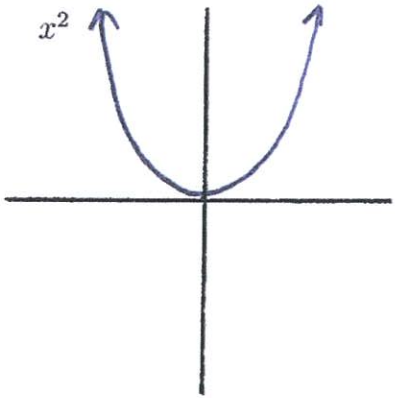
35.) 3



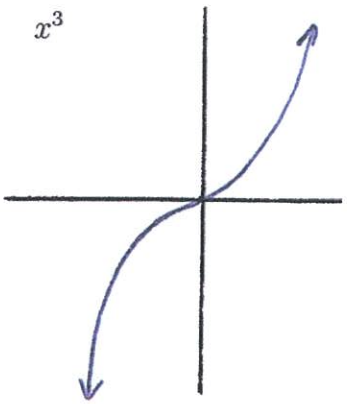
x



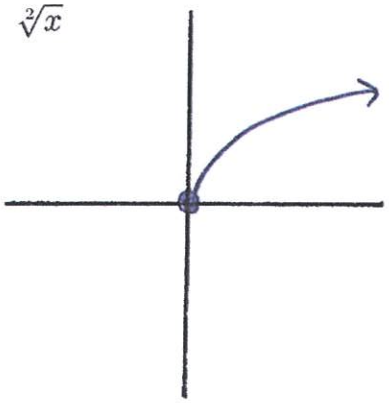
x^2



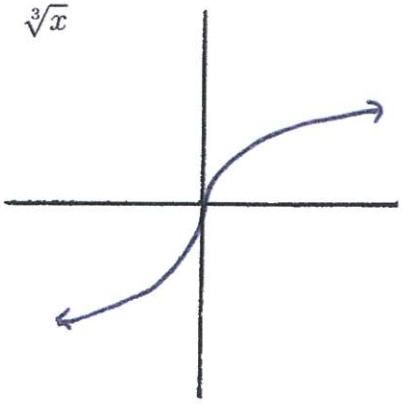
x^3



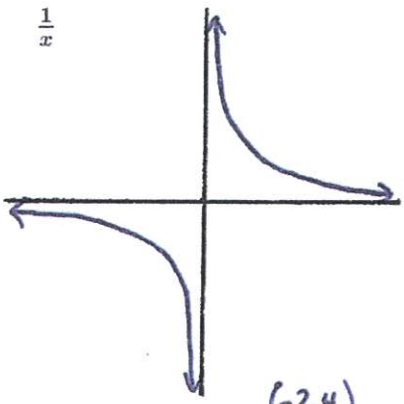
$\sqrt[2]{x}$



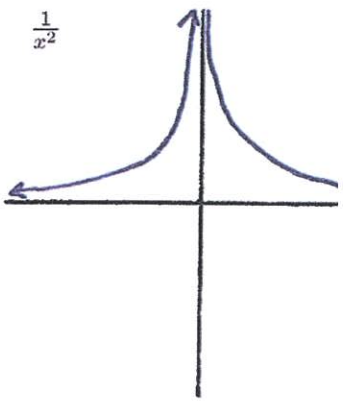
$\sqrt[3]{x}$



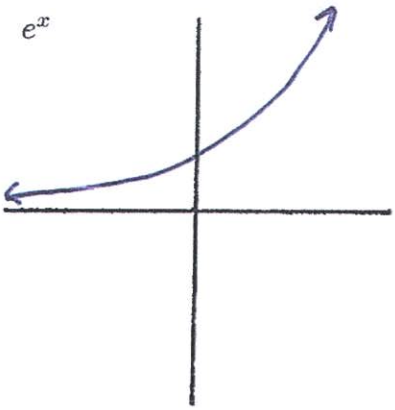
$\frac{1}{x}$



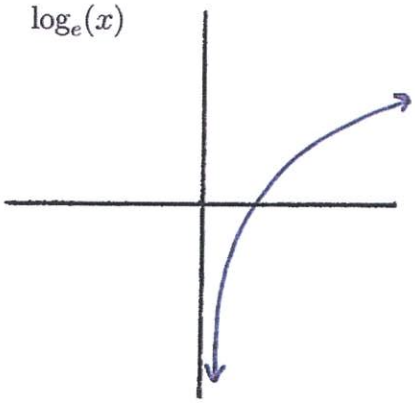
$\frac{1}{x^2}$



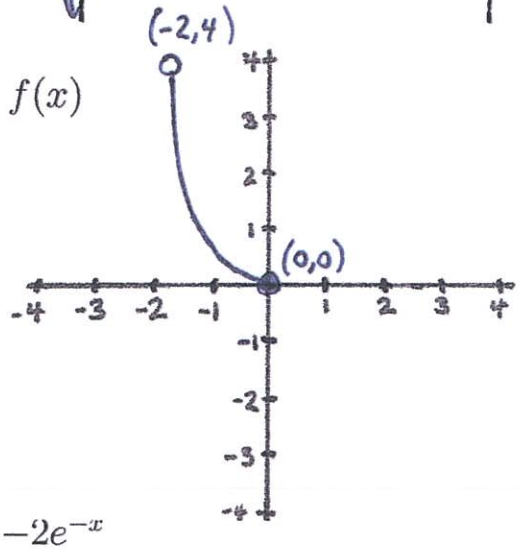
e^x



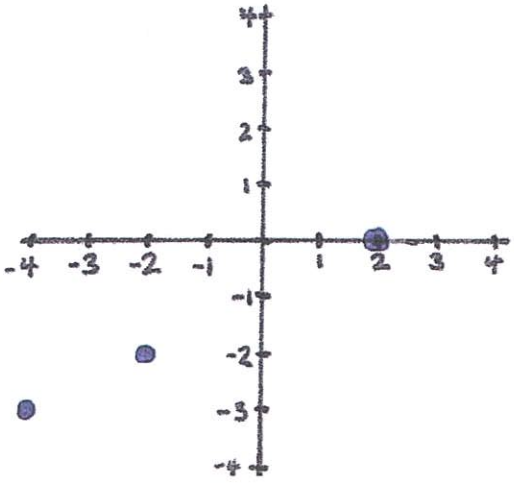
$\log_e(x)$



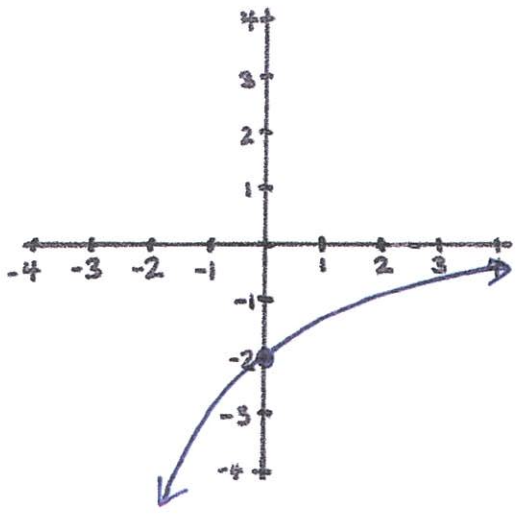
36.) $f(x)$



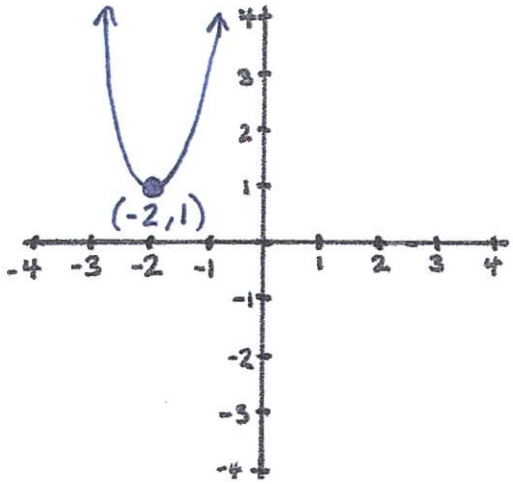
37.) $g(x)$



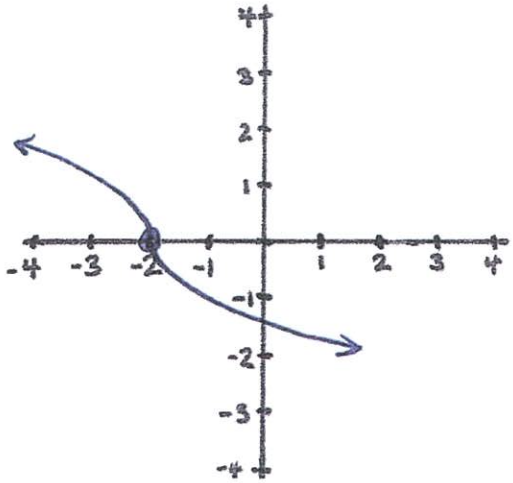
38.) $-2e^{-x}$



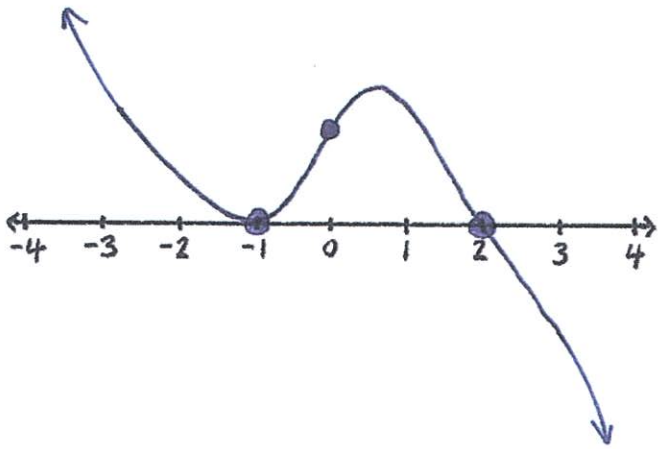
39.) $4(x+2)^2+1$



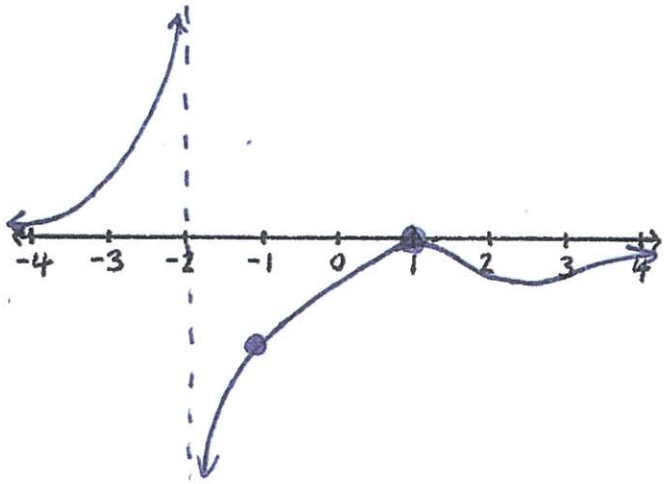
40.) $-\sqrt[3]{x+2}$



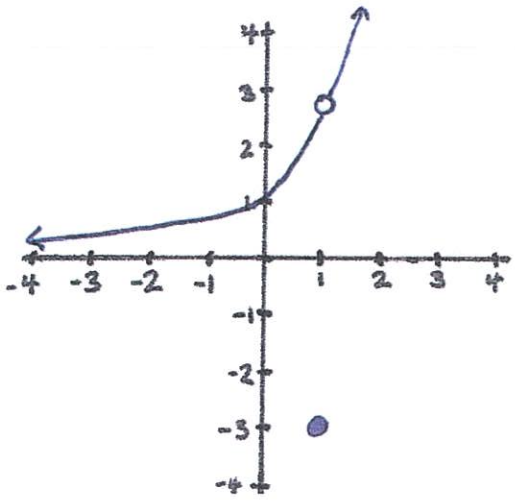
41.) $p(x)$



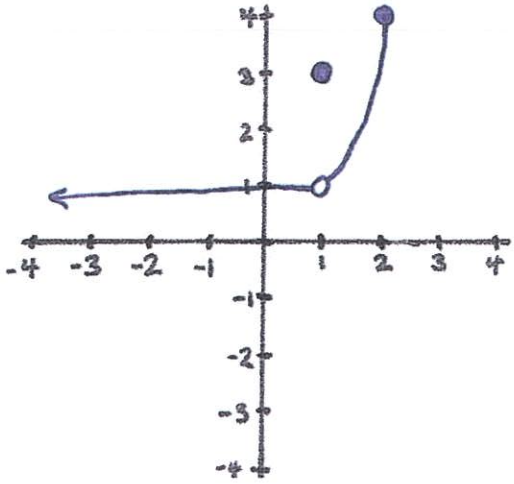
42.) $r(x)$



43.) $h(x)$



44.) $m(x)$



Practice Final Exam

Discrete math

1.) Find $\sum_{i=1}^{4}(i^2-1)$

$(1^2-1)+(2^2-1)+(3^2-1)+(4^2-1) = 0+3+8+15$

$= 26$

2.) Find $\sum_{i=1}^{\infty}\frac{2}{6^i}$

$\frac{\frac{2}{6}}{1-\frac{1}{6}} = \frac{2/6}{5/6} = (2/6)(6/5) = 2/5$

3.) What is the 61st term of the sequence $7, 11, 15, 19, \ldots$?

$7+60(4) = 7+240 = 247$

4.) What's the 57th term of $-3, 6, -12, 24, \ldots$?

$(-3)(-2)^{56}$

5.) What's the sum of the first 60 terms of the sequence $3, 5, 7, 9, \ldots$?

$a_{60} = 3+(59)2$

$= 3+118$

$= 121$

$\frac{60}{2}(a_1+a_{60}) = 30(3+121)$

$= 30(124)$

$= 3{,}720$

6.) Suppose a set A contains 243 objects. How many 92 object subsets of A are there?

$$\binom{243}{92}$$

7.) How many ways are there to choose and order 49 objects from a collection of 304 objects?

$$\frac{304!}{(304-49)!} = \frac{304!}{255!}$$

8.) How many different ways are there to order 93 different objects?

$$93!$$

9.) You're decorating a room by choosing a color to paint the walls with and a color of carpet to use for the floor. You have 6 different colors of paint to choose from for the walls, and 11 different colors of carpet to choose from for the floor. How many different wall and floor color combinations could you create?

$$6(11) = 66$$

10.) Write $\binom{9}{3}$ as an integer in standard form.

$$\frac{9!}{3!6!} = \frac{9\cdot8\cdot7\cdot6!}{3!6!} = \frac{9\cdot8\cdot7}{3!} = \frac{9\cdot8\cdot7}{3\cdot2\cdot1} = 3\cdot4\cdot7$$

$$= 84$$

****

Algebra

11.) Write $(\frac{27}{8})^{-\frac{2}{3}}$ as a rational number in standard form.

$$\left(\frac{27}{8}\right)^{-2/3} = \left(\frac{8}{27}\right)^{2/3} = \left(\frac{\sqrt[3]{8}}{\sqrt[3]{27}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

12.) Write $7^{-4}\sqrt[2]{7^9}(7^{\frac{1}{2}})^3$ as an integer in standard form.

$$7^{-4}\,7^{9/2}\,7^{3/2} = 7^{-8/2+9/2+3/2} = 7^2 = 49$$

13.) If $a \neq 0$, then what is a^0 as an integer in standard form? (Notice that this is asking for the y-intercept of the graph of a^x.)

1

14.) If $a > 0$, what is $\log_a(1)$ as an integer in standard form? (The answer has something to do with the x-intercept of the graph of $\log_a(x)$.)

0

15.) Write $\log_3\left(\sqrt[5]{\frac{1}{81}}\right)$ as an integer in standard form.

$$\log_3\left(3^{-\frac{4}{5}}\right) = -\frac{4}{5}$$

16.) Find x where $x^3(\frac{1}{2}x+3)^3 = 8$.

$$\left[x(\tfrac{1}{2}x+3)\right]^3 = 8$$

$$x(\tfrac{1}{2}x+3) = \sqrt[3]{8} = 2$$

$$\tfrac{1}{2}x^2+3x = 2$$

$$\tfrac{1}{2}x^2+3x-2=0$$

discriminant: $3^2-4(\tfrac{1}{2})(-2) = 9+4 = 13$

$$x = \frac{-3\pm\sqrt{13}}{2(\frac{1}{2})} = -3\pm\sqrt{13}$$

17.) Find x where $2\left(\frac{e^{2x}}{e^{x+3}}\right)+5=7$.

$$2e^{2x-(x+3)} = 2$$

$$e^{x-3} = 1$$

$$x-3 = \log_e(1) = 0 \implies x = 3$$

18.) Find x where $4\log_e(x)+\log_e(x^3)+8=11$.

$$\log_e(x^4)+\log_e(x^3) = 3$$

$$\log_e(x^4x^3) = 3$$

$$\log_e(x^7) = 3 \implies x^7 = e^3 \implies x = \sqrt[7]{e^3}$$

19.) Find $g\circ f(x)$ if $f(x)=x+2$ and $g(x)=x^2$.

$$gof(x) = g(x+2) = (x+2)^2$$

20.) Find the inverse of $g(x) = 7\log_e(x+3)$.

$$y = 7\log_e(x+3)$$

$$\frac{y}{7} = \log_e(x+3)$$

$$e^{y/7} = x+3$$

$$e^{y/7}-3 = x \implies e^{y/7}-3 = g^{-1}(x)$$

21.) What is the implied domain of $f(x) = x^2 - 2x + \log_e(3 - 7x)$?

$3 - 7x > 0 \Rightarrow 3 > 7x \Rightarrow \frac{3}{7} > x$.

Implied domain: $(-\infty, \frac{3}{7})$

22.) What is the implied domain of $g(x) = \frac{x^3}{2} - 7\sqrt[3]{x-4}$?

$\mathbb{R}$

23.) Find $\dfrac{x^3 - 3x^2 - 5x + 14}{x^2 - 4}$

$$
\begin{array}{r}
x - 3 \\
x^2 - 4 \;\overline{)\; x^3 - 3x^2 - 5x + 14} \\
-(x^3 \qquad - 4x) \\
\hline
-3x^2 - x + 14 \\
-(-3x^2 \qquad + 12) \\
\hline
-x + 2
\end{array}
$$

24.) Complete the square: Write $-2x^2 - 4x - 5$ in the form $\alpha(x+\beta)^2 + \gamma$ where $\alpha, \beta, \gamma \in \mathbb{R}$.

$$-2\left(x + \frac{-4}{2(-2)}\right)^2 - 5 - \frac{(-4)^2}{4(-2)} = -2(x+1)^2 - 5 + 2$$
$$= -2(x+1)^2 - 3$$

25.) How many roots does $2x^2 - 3x + 4$ have?

$(-3)^2 - 4(2)(4) = 9 - 32 < 0$, so no roots.

26.) Find a root of $x^3 + 2x^2 - x + 6$.

Factors of 6: $1, 6, -1, -6, 2, 3, -2, -3$

$(-3)^3 + 2(-3)^2 - (-3) + 6 = -27 + 18 + 3 + 6 = -27 + 27 = 0$

so -3 is a root.

27.) Completely factor $-2x^3 + 2x + 12$. (Hint: 2 is a root.)

2	-2	0	2	12
		-4	-8	-12
	-2	-4	-6	0

Discriminant of $-2x^2 - 4x - 6$:

$(-4)^2 - 4(-2)(-6) = 16 - 48 < 0$,

so $-2x^2 - 4x - 6$ has no roots.

$-2x^3 + 2x + 12$

$(x-2)$ $(-2x^2 - 4x - 6)$

(-2) $(x^2 + 2x + 3)$

28.) $|x - y|$ is the distance between which two numbers?

x and y

29.) Solve for x if $|3x - 2| < 4$.

$-4 < 3x - 2 < 4$

$-2 < 3x < 6$

$-\frac{2}{3} < x < 2$

****

Linear algebra

30.) What's the determinant of the matrix below?

$$\begin{pmatrix} 2 & -3 \\ 1 & -5 \end{pmatrix}$$

$2(-5) - (-3)(1) = -10 + 3 = -7$

31.) Find the product

$$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1\cdot 1 + 0\cdot 0 & 2\cdot 1 + 0\cdot 1 \\ 1\cdot 3 + 1\cdot 0 & 2\cdot 3 + 1\cdot 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$$

32.) What's the inverse of the matrix below?

$$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

$\det \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} = 3 - 8 = -5$

$$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}^{-1} = \frac{1}{-5} \begin{pmatrix} 3 & -4 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{pmatrix}$$

33.) Write the following system of three linear equations in three variables as a matrix equation

$$\begin{aligned} 2x - y + z &= 2 \\ y + 2z &= 1 \\ -x + y - z &= 0 \end{aligned}$$

$$\begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

34.) Solve for x, y, and z if

$$\begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ -4 & 5 & 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ -4 & 5 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2\cdot 1 + 2(-1) + 1\cdot 0 \\ (-1)(-2) + 2\cdot 2 + 1\cdot 1 \\ (-2)(-4) + 2\cdot 5 + 2\cdot 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 7 \\ 20 \end{pmatrix}$$

****

Graphs

35.) Graph the following functions: 3, x, x^2, x^3, $\sqrt[2]{x}$, $\sqrt[3]{x}$, $\frac{1}{x}$, $\frac{1}{x^2}$, e^x, $\log_e(x)$.

36.) Graph $f : (-2, 0] \to \mathbb{R}$ where $f(x) = x^2$.

37.) Graph $g : \{-4, -2, 2\} \to \mathbb{R}$ where $g(x) = \frac{1}{2}x - 1$.

$\frac{1}{2}(-4) - 1 = -2 - 1 = -3$

$\frac{1}{2}(-2) - 1 = -1 - 1 = -2$

$\frac{1}{2}(2) - 1 = 1 - 1 = 0$

38.) Graph $-2e^{-x}$ and label its y-intercept.

Start with graph of e^x: flip over x-axis; flip over y-axis

y-int: $-2e^{-0} = -2$

39.) Graph $4(x+2)^2 + 1$ and label its vertex.

graph $4x^2$ then shift left 2, up 1.

40.) Graph $-\sqrt[3]{x+2}$ and label its x-intercept.

Start with graph of $\sqrt[3]{x}$: flip over x-axis; shift left 2.

41.) Graph $p(x)$. (Label all x-intercepts.)

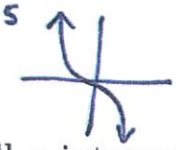
$$p(x) = -2(x+1)(x+1)(x-2)(x^2+1)$$

x-int: $-1, 2$

in between: $p(0) = -2(0+1)(0+1)(0-2)(0^2+1)$

$- \quad + \quad + \quad - \quad + \quad > 0$

leading term: $-2x^5$



42.) Graph $r(x)$ (Label all x-intercepts and all vertical asymptotes.)

$$r(x) = \frac{-3(x-1)(x-1)}{4(x+2)(x^2+1)}$$

vert. asymp.: -2

x-int: 1

in-between: $r(-1) = \dfrac{-3(-1-1)(-1-1)}{4(-1+2)((-1)^2+1)}$ $\qquad \dfrac{- \ - \ -}{+ \ + \ +} < 0$

quotient of leading terms: $\dfrac{-3x^2}{4x^3} = -\frac{3}{4}\,\frac{1}{x}$

43.) Graph

$$h(x) = \begin{cases} e^x & \text{if } x \neq 1; \\ -3 & \text{if } x = 1. \end{cases}$$

44.) Graph

$$m(x) = \begin{cases} 1 & \text{if } x \in (-\infty, 1); \\ 3 & \text{if } x = 1; \\ x^2 & \text{if } x \in (1, 2]. \end{cases}$$