Last Name:______________________ First Name:______________________

1.) True

2.) False

3.) False

4.) True

5.) True

6.) True

7.) True

8.) False

9.) 5

10.) 2

11.) $f^{-1}(y) = \frac{y^3-4}{2}$

12.) $[-\frac{1}{5}, \infty)$

13.) 0

14.) $2x^2-3x+1+\frac{-3x+3}{2x^2-1}$

15.) $4x^2+8x+11+\frac{28}{x-2}$

16.) 2

17.) $2(x-1)^2+3$

18.) 0

19.) $\frac{3+\sqrt{13}}{2}$ and $\frac{3-\sqrt{13}}{2}$

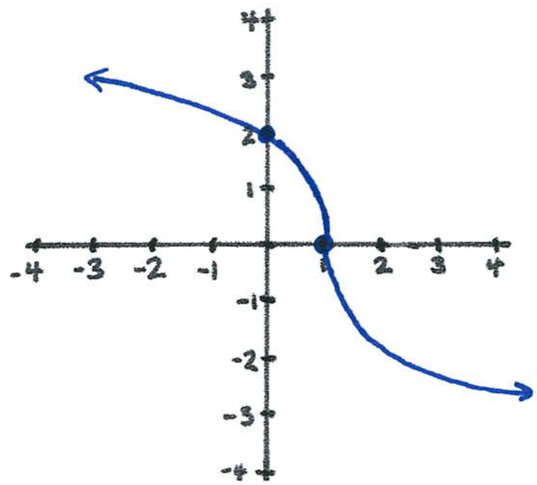
20.) -2

21.) $-2(x-3)(x^2+1)$

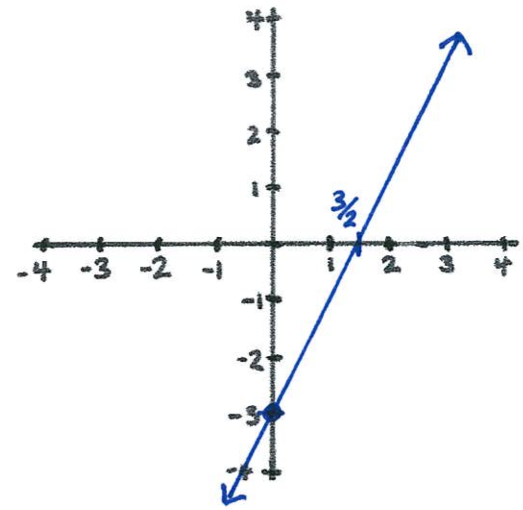
22.) $2(x+7)(x-1)(x-\frac{1}{2})$

23.) $x+2$, $x-3$

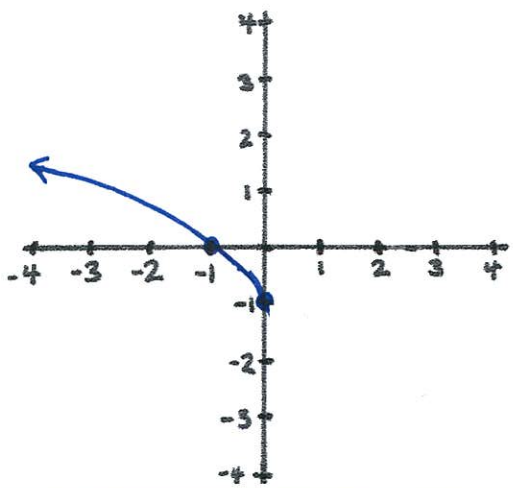
24.) $-2\sqrt[3]{x-1}$



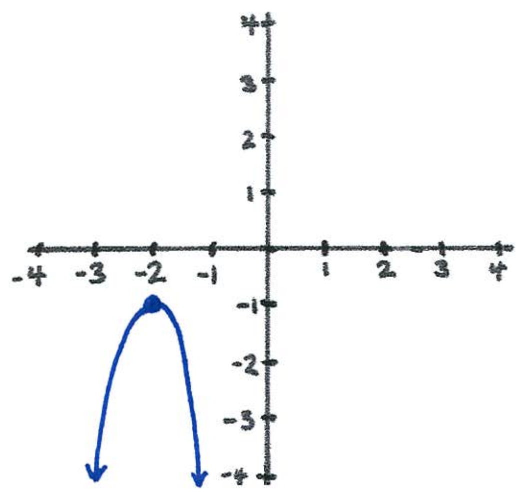
26.) $2x-3$



25.) $\sqrt{-x}-1$



27.) $-3(x+2)^2-1$



Answers that would be marked wrong on the exam include:

14.) $2x^2-3x+1 \quad R \quad -3x+3$

14.) $2x^2-3x+1-\frac{3x+3}{2x^2-1}$

15.) $4x^2+8x+11 \quad R \quad 28$

19.) $\frac{6 \pm \sqrt{52}}{4}$

19.) $\frac{6 \pm 2\sqrt{13}}{4}$

11.) $f^{-1}(y)=\frac{x^3-4}{2}$

11.) $f^{-1}(x)=\frac{y^3-4}{2}$

However, writing $f^{-1}(x)=\frac{x^3-4}{2}$ is a correct answer for #11.

Second Practice Exam

True/False

For #1-8 write the entire word "True" or the entire word "False".

1.) $a(b+c) = ab + ac$

2.) $(x+y)^n = x^n + y^n$ False: $(1+1)^2 \neq 1^2 + 1^2$

3.) $\sqrt[n]{x+y} = \sqrt[n]{x} + \sqrt[n]{y}$ False: $\sqrt[2]{1+1} \neq \sqrt[2]{1} + \sqrt[2]{1}$

4.) $(xy)^n = x^n y^n$

5.) $\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$

6.) $(\frac{x}{y})^n = \frac{x^n}{y^n}$

7.) $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

8.) $-x^4 + x^3 + 17x^2 - 3x + 4$ has 7 roots. False: (degree) $\geq$ (# of roots)

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Algebra

9.) Find x where $(7-x)^3 + 4 = 12$.

$(7-x)^3 = 12-4 = 8$

$7-x = \sqrt[3]{8} = 2$

$-x = 2-7 = -5$

$x = 5$

10.) If $g(x)$ is an invertible function, and $g(2) = 7$, then what is $g^{-1}(7)$?

$2 = g^{-1}(7)$

11.) Find the inverse of $f(x) = \sqrt[3]{2x+4}$. (You can check your answer by seeing if $f^{-1} \circ f(x) = x$.)

$(f(x) \mapsto y)$ $\qquad$ $y = \sqrt[3]{2x+4}$

(Solve for x) $\qquad$ $y^3 = 2x+4$

$y^3 - 4 = 2x$

$\frac{y^3-4}{2} = x$

$(x \mapsto f^{-1}(y))$ $\qquad$ $\frac{y^3-4}{2} = f^{-1}(y)$

12.) What is the implied domain of $g(x) = 2\sqrt[2]{5x+1} - 23$? (Your answer should be an interval.)

We can't take an even root of a negative number, so $5x+1 \geq 0$.

Hence, $5x \geq -1$ which implies $x \geq -1/5$.

Implied domain: $[-1/5, \infty)$

13.) Suppose that $a \neq 0$ and that $b^2 - 4ac \geq 0$. Write the following number as an integer in standard form:

$$a\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)^2 + b\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right) + c$$

The quadratic formula says that $\frac{-b+\sqrt{b^2-4ac}}{2a}$ is a root of ax^2+bx+c. That means that the number above is 0.

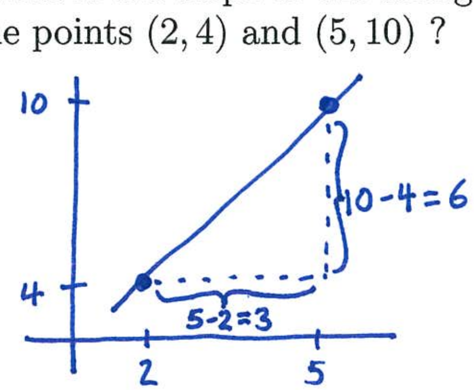
14.) Find $\dfrac{4x^4 - 6x^3 + 2}{2x^2 - 1}$

$$
\begin{array}{r|l}
 & \;\; 2x^2 - 3x + 1 \\
\hline
2x^2 - 1 & 4x^4 - 6x^3 \qquad\qquad + 2 \\
 & -(4x^4 \qquad - 2x^2 \qquad\quad) \\
\hline
 & \qquad -6x^3 + 2x^2 \qquad + 2 \\
 & \quad -(-6x^3 \qquad + 3x \quad) \\
\hline
 & \qquad\qquad 2x^2 - 3x + 2 \\
 & \qquad\quad -(2x^2 \qquad - 1) \\
\hline
 & \qquad\qquad\qquad -3x + 3
\end{array}
$$

15.) Find $\dfrac{4x^3 - 5x + 6}{x - 2}$

$$
\begin{array}{r|rrrr}
2 & 4 & 0 & -5 & 6 \\
 & & 8 & 16 & 22 \\
\hline
 & 4 & 8 & 11 & \vdots\, 28
\end{array}
$$

16.) What is the slope of the straight line in $\mathbb{R}^2$ that passes through the points $(2, 4)$ and $(5, 10)$?



$$\frac{10-4}{5-2} = \frac{6}{3} = 2$$

17.) Complete the square: Write $2x^2 - 4x + 5$ in the form $\alpha(x+\beta)^2 + \gamma$ where $\alpha, \beta, \gamma \in \mathbb{R}$.

$$2\left(x + \frac{-4}{2\cdot 2}\right)^2 + 5 - \frac{(-4)^2}{4(2)} = 2(x-1)^2 + 3$$

18.) How many roots does $2x^2 - 6x + 10$ have?

Discriminant: $(-6)^2 - 4(2)(10) = 36 - 80 < 0$,

so no roots

19.) Find the roots of $-2x^2 + 6x + 2$

Discriminant: $6^2 - 4(-2)(2) = 36 + 16 = 52$,

so 2 roots.

$$\sqrt{52} = \sqrt{2\cdot 2\cdot 13} = 2\sqrt{13}$$

Roots: $\dfrac{-6 \pm \sqrt{52}}{2(-2)} = \dfrac{-6 \pm 2\sqrt{13}}{2(-2)} = \dfrac{3 \pm \sqrt{13}}{2}$

20.) Find a root of $-3x^3 - 7x^2 - 4x - 4$

Factors of -4 are $1, -1, 2, -2, 4, -4$. One of these numbers will probably be a root.

Check to see that

$$-3(-2)^3 - 7(-2)^2 - 4(-2) - 4 = -3(-8) - 7(4) - 4(-2) - 4$$
$$= 24 - 28 + 8 - 4$$
$$= 0$$

so -2 is a root.

21.) (2 points) Completely factor $-2x^3 + 6x^2 - 2x + 6$. (Hint: 3 is a root.) (Your answer should be a product of a constant and maybe some linear and quadratic polynomials that have leading coefficients equal to 1, and such that any of the quadratics in the product have no roots.)

$$\begin{array}{r|rrrr} 3 & -2 & 6 & -2 & 6 \\ & & -6 & 0 & -6 \\ \hline & -2 & 0 & -2 & 0 \end{array}$$

so $-2x^3+6x^2-2x+6$

$(x-3)$ $\quad$ $(-2x^2-2)$

Discriminant of $-2x^2-2$ is

$0^2-4(-2)(-2)=0-16<0$

so $-2x^2-2$ has no roots.

Thus, $-2x^2-2$ completely factors as $-2(x^2+1)$:

$-2x^3+6x^2-2x+6$

$(x-3)$ $\quad$ $(-2x^2-2)$

(-2) $\quad$ (x^2+1)

22.) (2 points) Completely factor $2x^3 + 11x^2 - 20x + 7$. (Hint: -7 is a root.) (Your answer should have the same form as described in #21)

$$\begin{array}{r|rrrr} -7 & 2 & 11 & -20 & 7 \\ & & -14 & 21 & -7 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

so $2x^3+11x^2-20x+7$

$(x+7)$ $\quad$ $(2x^2-3x+1)$

Discriminant of $2x^2-3x+1$ is

$(-3)^2-4(2)(1)=9-8=1$

so $2x^2-3x+1$ has 2 roots. They are:

$\dfrac{-(-3)+\sqrt{1}}{2(2)} = \dfrac{3+1}{4} = \dfrac{4}{4} = 1$, and

$\dfrac{3-1}{4} = \dfrac{2}{4} = \frac{1}{2}$. Thus, $2x^2-3x+1$

completely factors as $2(x-1)(x-\frac{1}{2})$

$2x^3+11x^2-20x+7$

$(x+7)$ $\quad$ $(2x^2-3x+1)$

(2) $\quad$ $(x-1)$ $\quad$ $(x-\frac{1}{2})$

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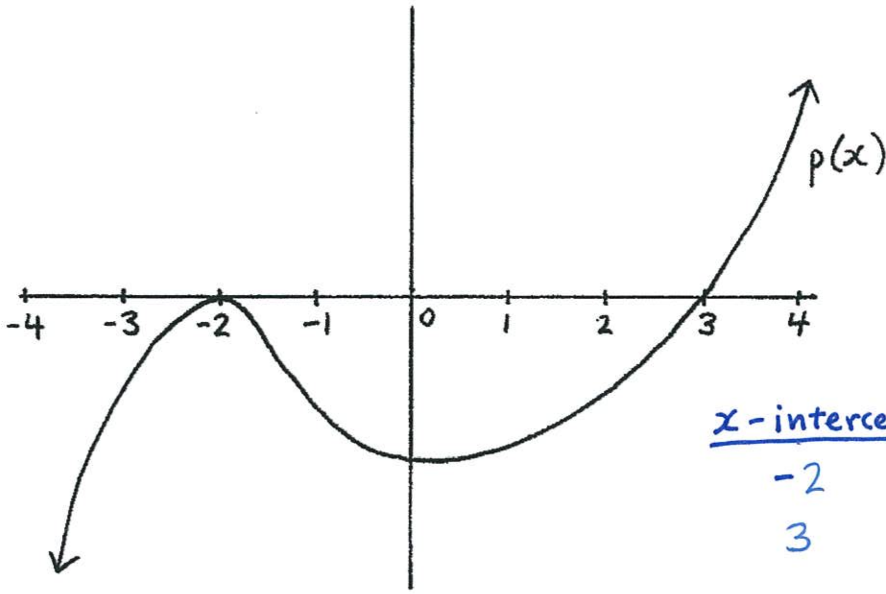
Graphs

23.) List all of the monic linear factors of $p(x)$ that you know of from the graph below.



x-intercepts	$\to$ factors
-2	$x+2$
3	$x-3$

24.) Graph $-2\sqrt[3]{x-1}$ and label its x- and y-intercepts. Graph $\sqrt[3]{x}$. Flip over x-axis. Stretch vertically. Shift right 1. x-intercept is solution to $-2\sqrt[3]{x-1}=0$. Namely, $x=1$. y-intercept is $-2\sqrt[3]{0-1}=2$

25.) Graph $\sqrt{-x}-1$ and label its x- and y-intercepts. Graph $\sqrt{x}$. Flip over y-axis. Shift down 1. x-intercept is solution to $\sqrt{-x}-1=0$, namely, $x=-1$. y-intercept is $\sqrt{-0}-1=-1$

26.) Graph $2x-3$ and label its x- and y-intercepts. $2x-3=2(x-\frac{3}{2})$ so root and x-intercept is $\frac{3}{2}$. y-intercept is $2(0)-3=-3$. Graph is a straight line through x- and y-intercepts.

27.) Graph $-3(x+2)^2-1$ and label its vertex.

Graph $-3x^2$. Shift left 2. Shift down 1.