1. Consider the sets $A = \{n \in \mathbb{N} : n \text{ is even }\}$, $B = \{n \in \mathbb{N} : |n| \leq 10\}$, and $C = \{3n + 1 : n \in \mathbb{Z}\}$.
   
   (a) List the elements of $A$, $B$, and $C$.
   
   (b) List the elements of $A \cap B$, $A \cup C$, $B \cup C$, $B \setminus A$, $A \setminus B$ and $Z \setminus C$.
   
   (c) Check directly that the sets $(A \cap B) \cup C$ and $(A \cup C) \cap (B \cup C)$ are equal.
   
   (d) Prove or disprove that the identity from part (c) holds for any three sets: that is, for any sets $R$, $S$, $T$, prove or disprove that
   
   $$(R \cap S) \cup T = (R \cup T) \cap (S \cup T).$$

2. Determine whether each of the identities about sets is true or false. If an identity is true, prove it is true either directly or using a membership table or Venn diagram. If an identity is false, provide a specific counter-example.
   
   (a) $A \setminus (A \setminus B) = B \setminus (B \setminus A)$
   
   (b) $(A \cup B) \cap C = A \cap (B \cup C)$
   
   (c) $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$
   
   (d) $C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B)$.
   
   (e) $A \setminus (B \setminus A) = \emptyset$.

3. Prove the following statements about sets.
   
   (a) If $A \setminus B = B \setminus A$, then $A = B$.
   
   (b) If $A \subset B$, then $B \setminus A = \emptyset$.

4. For each of the following functions from $Z$ to $Z$, determine if it is injective, surjective, both, or neither, and if it is increasing, decreasing or neither. Justify your conclusions.
   
   (a) $f(n) = \left\lfloor \frac{3n}{5} \right\rfloor$
   
   (b) $g(x) = |2x + 1|$
   
   (c) $h(k) = 5(3 - k) - 2$

5. Determine the range of the function $T : \mathbb{R} \to \mathbb{R}$ given by $T(x) = \lceil x \rceil - \lfloor x \rfloor$. (Recall that $\lceil x \rceil$ is the “ceiling of $x$”, which is the smallest integer $n \geq x$.)

6. Let $E = \{2, 4, 6, \ldots\}$ be the set of positive even integers and consider the function $f : \mathbb{N} \to E$ given by $f(n) = 4n - 2$ and the function $g : E \to \mathbb{N}$ given by $g(m) = m/2$.
   
   (a) Determine the domain, codomain, and formula for the composition $f \circ g$. Determine if it is injective, surjective, both, or neither.
   
   (b) Determine the domain, codomain, and formula for the composition $g \circ f$. Determine if it is injective, surjective, both, or neither.

7. Prove that the function $q : \mathbb{N} \to \mathbb{Z}$ given by $q(n) = (-1)^n \left\lfloor \frac{n}{2} \right\rfloor$ is a bijection.

8. Write down the first 5 terms of the sequence $\{a_n\}$ defined by $a_0 = 1$ and $a_n = \frac{1}{2}(a_{n-1} + 1)$.

9. Find three different sequences beginning with the terms $1, 2, 4$ whose terms are generated by a simple rule, formula, or recurrence relation.

10. Write down the first 5 terms the sequence $s_n$ given by

    $$s_n = \sum_{j=1}^{n} j^3.$$ 

    Do you notice a different pattern? Describe what pattern you see without proving that it is true in general.