1. (a) Prove or disprove the statement:
   If \( d \mid a \) and \( d \mid b \), then \( d \mid (a + b) \).
(b) Prove or disprove the statement:
   If \( d \mid (a + b) \), then \( d \mid a \) and \( d \mid b \).

2. (a) Use the Euclidean algorithm to find \( d = \gcd(225, 70) \).
(b) Use part (a) to find an integer solution to \( 225x + 70y = d \).

3. Consider the statement \( p(n) \) given by the equation
   \[
   1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n + 1)(2n + 1)}{6}.
   \]
   (a) Verify directly that \( p(n) \) is true for \( n = 1, 2, 3, 4 \).
(b) Use mathematical induction to prove that \( p(n) \) is true for all integers \( n \geq 1 \).

4. A PIN for an account consists of 5 characters composed of lower-case letters and numbers.
   (a) How many PINs are there that consist of all distinct characters?
   (b) How many PINs are there that have at least one number.