Some Review Problems
for Exam #3

1. Compute the following indefinite integrals:

   (a) $\displaystyle\int \frac{r^5 - \sqrt{r}}{r^3}\,dr$

   (b) $\displaystyle\int \sec^2(8y)\,dy$

   (c) $\displaystyle\int 3x^4\sqrt{x^5 - 2}\,dx$

   (d) $\displaystyle\int \frac{1}{w^3}\,\cos\frac{1}{w^2}\,dw$

2. Compute the following definite integrals:

   (a) $\displaystyle\int_{1/6}^{1/3} \cos(\pi z)\,dz$

   (b) $\displaystyle\int_{0}^{\pi/2} \sin v\,\cos v\,dv$

   (c) $\displaystyle\int_{1}^{2} \frac{3t^2 + 5t}{(4t^3 + 10t^2 - 3)^2}\,dt$

   (d) $\displaystyle\int_{1/4}^{1/2} \Big(4s + \pi\cos(\pi s)\Big)\,ds$

3. Find the particular solution $x(t)$ of the following differential equation, and compute $x(5)$.

$$\frac{dx}{dt} = \frac{2t}{3x^2}\,, \qquad x(1) = 3$$

4. Suppose a watermelon is thrown upward at 6 feet per second from the top of an 80-foot building.

   (a) Find the equation for the velocity $v(t)$ of the watermelon at all times and compute $v(1)$.

   (b) Find the equation for the position $s(t)$ of the watermelon at all times and compute $s(1)$.

   (c) When is the maximum height attained and what is the maximum height?

   (d) When does the watermelon hit the ground below and at what speed does it hit?

5. Suppose the acceleration of a particle moving along the x-axis is given by $a(t) = 4 - t$, where t is counted in seconds. Further suppose that the particle satisfies $v(1) = 5$ and $s(2) = 10$. Find the equations of the particle's velocity and position at all times.

6. Suppose the position (in thousands of miles) of an electron in a(n unorthodox) particle accelerator is given by $s(t) = \sqrt{t^2 + 16}$, where t is measured in seconds.

   (a) Find the average velocity of the particle over the time interval $[0, 3]$.

   (b) Find a time in between $t = 0$ and $t = 3$ at which the particle has instantaneous velocity equal the average velocity.

7. The Mean Value Theorem states that for a function $f(x)$ that is continuous on an interval $[a, b]$ and differentiable on the interval (a, b), there is at least one value $x = c$ between a and b such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

For the function $f(x) = x^2$ defined on the interval $[1, 6]$, find the value c guaranteed by the theorem.

8. Find the tangent line to the curve $y = \int_0^{1/x} \frac{1}{\sqrt{1-t^2}}\, dt$ at the point $(2, \pi/6)$.

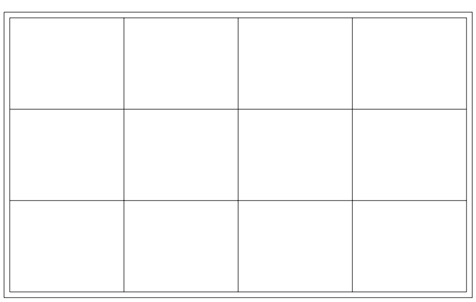
9. For $f(x) = \int_0^x t\sqrt{t+1}\, dt$, find intervals on which $f(x)$ is concave up and concave down.

10. Compute the integral $\int_0^5 |3x - 6|\, dx$ by graphing the function and finding the area under the curve using geometry.

11. Approximate the area under the curve $3x^2 + 1$ from $x = -1$ to $x = 2$ by dividing the interval $[-1, 2]$ into 6 pieces and using the right endpoints of the subintervals to determine the height of the rectangles.

12. Use the definite integral to find the area under the curve $y = \frac{1}{\sqrt{x}}$ between $x = 4$ and $x = 64$.

13. A city planner in a small town wants to put 12 rectangular blocks inside a rectangular region with a total area of $15,000,000$ square feet. The city does **not** have to maintain the roads that go around the outside of the rectangular region since they will be state highways. What should the dimensions of one city block be in order to minimize the amount of road to be maintained by the city.



14. Find the point (x, y) on the curve $y = \sqrt{9 - x}$ in the first quadrant such that the rectangle formed has maximal area. Justify (by using the first derivative test or by checking the endpoints of the domain) that the point found yields the maximal area.