

Practice Final Exam

Instructions: Answer all twelve of the following questions.

1. What is the value of the integral $\int_{-2}^2 t \sin(\pi t^2) dt$? Justify your answer.
2. Find the equation of the tangent line at the point $(1, 1)$ to the curve $y^2 + yx = 2x^5$.
3. Let $p(w) = 4w^2 - 2w + 5$. Compute $p'(w)$ using the limit definition of the derivative:

$$p'(w) = \lim_{h \rightarrow 0} \frac{p(w+h) - p(w)}{h}$$

4. Let $g(y) = \frac{\cos \pi y}{\sqrt{y^2 + 3}}$. Compute $g'(y)$.
5. Consider the graph of the function $f(x) = x^3 - 2x^2 - 4x + 8$.
 - (a) Find the y -intercept and the x -intercepts of $y = f(x)$.
 - (b) On what interval(s) is $f(x)$ increasing? decreasing? Find the point(s) (x, y) where $f(x)$ has a local maximum or minimum.
 - (c) On what interval(s) is $f(x)$ concave up? concave down? Find the point(s) (x, y) where $f(x)$ has a point of inflection.
 - (d) Draw a graph of the function $y = f(x)$.

6. Let $\gamma(x) = \int_1^{\cos(x)} -\frac{1}{t} dt$. Find $\gamma''(\pi/4)$.

7. Compute the following limits:

- (a) $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x})$

- (b) $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{2x + 1}$

8. Find the area under the curve $y = 16x(\pi x^2 + 8)^{5/3}$ between $x = 1$ and $x = 4$.
9. Solve the following differential equation:

$$\frac{dw}{dx} = w^2 \sin x \quad w(0) = \frac{1}{2}$$

10. A 128-foot-tall robot skeleton throws a robot vacuum straight up in the air at 96 feet per second. Assume the vacuum is released at the same height as the skeleton. How high does the robot vacuum go? How fast is it going when it hits the ground? (Recall that the acceleration due to gravity is 32 feet per second per second.)
11. An airplane flies directly over Claudius at a speed of 500 miles per hour and at an altitude of 3 miles. How fast is the distance between Claudius and the airplane increasing when the plane is 4 miles past Claudius?
12. Find the dimensions of the rectangle inscribed in an equilateral triangle of height = 10 and base = 16 that has the maximal area of all such rectangles.