Exam #1 Solutions

1. Let $f(x) = 3x - 5$ and $g(x) = x^2 + 4$. Is the function $f\big(g(x)\big)$ even, odd or neither? Justify your answer.

   First of all, $g(x)$ is even since

   $$g(-x) = (-x)^2 + 4 = x^2 + 4 = g(x).$$

   Therefore,

   $$f\big(g(-x)\big) = f\big(g(x)\big)$$

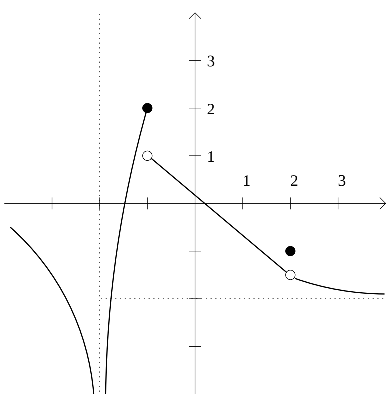
   so $f\big(g(x)\big)$ is also even.

2. Consider the function $r(t) = -10 + 25\sin\left(\frac{t}{8}\right)$. What is the amplitude of $r(t)$? What is the period of $r(t)$?

   The amplitude of $r(t)$ is 25. The period of $r(t)$ is

   $$\text{period} \;=\; \frac{2\pi}{1/8} = 2\pi \cdot 8 = 16\pi$$

3. Consider the graph of the function $y = f(x)$ given below.



   Compute the following limits: ($\pm\infty$ is a valid limit; write DNE if the limit doesn't exist)

   $\lim_{x\to -2} f(x) = -\infty$

   $\lim_{x\to 2^-} f(x) = -1.5$

   $\lim_{x\to -1^+} f(x) = 1$

4. Compute the following limit:

   $$\lim_{x\to 0} \frac{\sin 2x \cos 3x}{4x} = \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{\cos 3x}{2} = 1 \cdot \frac{1}{2} = \frac{1}{2}$$

5. Let $\ell(x) = \dfrac{x^2+5x-6}{4x^2-4}$. Compute the following limits:

(a) $\lim\limits_{x\to\infty} \ell(x) = \lim x\infty \dfrac{x^2+5x-6}{4x^2-4} = \dfrac{1}{4}$

(b) $\lim\limits_{x\to 1} \ell(x) = \lim\limits_{x\to 1} \dfrac{(x-1)(x+6)}{4(x-1)(x+1)} = \lim\limits_{x\to 1} \dfrac{x+6}{x+1} = \dfrac{7}{2}$

6. For the function $m(t)$ given below, determine what value of the constant A will make $m(t)$ a continuous function.

$$m(t) = \begin{cases} t^3 - 1 & , \quad t \le -1 \\ 3t + A & , \quad t > -1 \end{cases}$$

In order for $m(t)$ to be continuous, we need that $\lim\limits_{t\to 1^+} m(t) = \lim\limits_{t\to 1^-} m(t)$.

$$\lim_{t\to 1^+} m(t) = \lim_{t\to 1} 3t + A = 3 + A$$

$$\lim_{t\to 1^-} m(t) = \lim_{t\to 1} t^3 - 1 = 0$$

So $m(t)$ is continuous for $A = -3$.

7. Compute the slope of the line passing through the points $(-1, 3)$ and $(7, -1)$.

$$\text{slope} \;= \frac{y_2 - y_1}{x_2 - x_1} = -1 - 37 - (-1) = \frac{-4}{8} = -\frac{1}{2}$$

8. Compute the following limits:

$$\begin{aligned}
\lim_{x\to\infty} \left(\sqrt{x^2+2x} - x\right) &= \lim_{x\to\infty} \left(\sqrt{x^2+2x} - x\right) \cdot \frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x} \\
&= \lim_{x\to\infty} \frac{x^2+2x-x^2}{\sqrt{x^2+2x}+x} \\
&= \lim_{x\to\infty} \frac{2x}{\sqrt{x^2+2x}+x} \cdot \frac{1/x}{1/x} \\
&= \lim_{x\to\infty} \frac{2}{\sqrt{1+2/x}+1} \\
&= 1
\end{aligned}$$

$$\begin{aligned}
\lim_{x\to-\infty} \frac{8x+5}{\sqrt{2x^2+3x-1}} &= \lim_{x\to-\infty} \frac{8x+5}{\sqrt{2x^2+3x-1}} \cdot \frac{1/x}{-1/\sqrt{x^2}} \\
&= \lim_{x\to-\infty} \frac{8+5/x}{-\sqrt{2+3/x-1/x^2}} \\
&= \frac{8}{-\sqrt{2}} \\
&= -4\sqrt{2}
\end{aligned}$$