

**Homework #8**

Instructions: Answer the following questions on a **separate sheet of paper**.

1. Suppose that the population of ants in Pollyanna's ant colony is decreasing at a rate of 3% per year.

(a) If she has 1500 ants in 2008, how many will there be in 2060?

Use  $Q = Q_0(1 + r)^t$  with  $Q_0 = 1500$  and  $r = -.03$ .

$$Q = 1500(.97)^t$$

To solve for the number of ants in 2060, use  $t = 52$  since  $t = 0$  corresponds to 2008.

$$Q = 1500(.97)^{52} = 307.77$$

so there are about 308 ants at the beginning of 2060.

(b) If she has 1500 ants in 2008, how many were there in 1999?

To solve for the number of ants in 1999, use  $t = -9$ .

$$Q = 1500(.97)^{-9} = 1973.08$$

so there were about 1973 ants at the beginning of 1999.

(c) In what year will the number of ants drop below 1000?

To solve for the year in which the population is 1000, use  $Q = 1000$ .

$$\begin{aligned} 1000 &= 1500(.97)^t \\ \frac{1000}{1500} &= .97^t \\ \log \frac{2}{3} &= \log (.97^t) \\ \log \frac{2}{3} &= t \log .97 \\ \frac{\log \frac{2}{3}}{\log .97} &= t \\ 13.31 &= t \end{aligned}$$

so about a third of the way through 2021 there will only be 1000 ants.

(d) What is the half-life of the population of Pollyanna's ant colony?

We can just calculate  $T_{\text{half}}$  by

$$T_{\text{half}} = \frac{\log(.5)}{\log(.97)} = 22.76$$

so the number of ants is cut in half every 22.76 years.

2. Suppose that the population of ants in Felix's ant colony is increasing at a rate of 6% per day.

(a) If Felix buys 20 ants, how many will he have in 3 weeks?

We'll use  $Q = Q_0(1 + r)^t$  where  $t$  is time in days,  $t = 0$  corresponds to now,  $Q_0 = 20$ , and  $r = .06$ .

$$Q = 20(1.06)^t$$

To find how many ants Felix will have in 3 weeks, we use  $t = 21$ , since  $t$  is counted in days.

$$Q = 20(1.06)^{21} = 67.99$$

so there will be about 68 ants after 3 weeks.

(b) When will Felix's ant colony reach a population of 100?

To find when the population will reach 100, we use  $Q = 100$  and solve for  $t$ .

$$\begin{aligned} 100 &= 20(1.06)^t \\ 5 &= 1.06^t \\ \log 5 &= \log (1.06^t) \log 5 &= t \log 1.06 \\ \frac{\log 5}{\log 1.06} &= t \\ 27.62 &= t \end{aligned}$$

so the population will reach 100 after 27.62 days.

(c) How often does the number of ants double?

$$T_{\text{double}} = \frac{\log 2}{\log 1.06} = 11.896$$

so the population doubles every 11.896 days.

3. Suppose that Pollyanna has a viral infection that is being treated by some herbal remedy, and that the number of viruses is cut in half every 3 days.

(a) What percentage of the original viruses are left after 2 weeks?

We'll use  $Q = Q_0 \left(\frac{1}{2}\right)^{t/T_{\text{half}}}$  with  $t$  in days,  $t = 0$  corresponds to now and  $T_{\text{half}} = 3$ . We can use  $Q_0 = 100$  and  $Q$  will be the percentage of viruses left after  $t$  days.

$$Q = 100(.5)^{t/3}$$

To find the percentage of viruses left after 2 weeks, we'll use  $t = 14$ .

$$Q = 100(.5)^{14/3} = 3.937$$

so there are only 3.937% of the original viruses left after 2 weeks.

(b) After how many days will the number of viruses be at 16% of the original number?

To find the when the percentage reaches 16%, use  $Q = 16$  and solve for  $t$ .

$$16 = 100(.5)^{t/3}$$

$$.16 = (.5)^{t/3}$$

$$\log .16 = \log \left( (.5)^{t/3} \right)$$

$$\log .16 = \frac{t}{3} \log .5$$

$$3 \frac{\log .16}{\log .5} = t$$

$$7.93 = t$$

so it takes nearly 8 days to reach 16%.

(c) What is the daily decay rate of the viruses? (Hint, use  $t = 1$  day.)

$$Q = 100(.5)^{1/3} = 79.37$$

so after one day, the number of viruses dropped 20.63%.

4. Suppose that Felix has a viral infection that just won't go away no matter what he tries, and that the number of viruses doubles every 25 weeks.

- (a) If Felix's infection consists of a modest 22 viruses right now, how many viruses will he have in 5 weeks?

We'll use  $Q = Q_0(2)^{t/T_{\text{double}}}$ . We'll measure  $t$  in weeks with  $t = 0$  corresponding to right now with  $T_{\text{double}} = 25$ , with  $Q_0 = 22$ .

$$Q = 22(2)^{t/25}$$

To find the number of viruses after 5 weeks, we use  $t = 5$ .

$$Q = 22(2)^{5/25} = 25.27$$

so after 5 weeks, there are about 25 viruses.

- (b) If Felix's infection consists of 22 viruses right now,, how many viruses were infecting him 1 week ago?

To find the number a week ago, use  $t = -1$ .

$$Q = 22(2)^{-1/25} = 21.398$$

so last week he had roughly 21 viruses.

- (c) How many weeks will pass until the infection reaches 10,000 viruses?

To find when there are 10,000 viruses, use  $Q = 10,000$  and solve for  $t$ .

$$\begin{aligned} 10,000 &= 22(2)^{t/25} \\ \frac{10,000}{22} &= 2^{t/25} \\ \log \frac{10,000}{22} &= \log \left( 2^{t/25} \right) \\ \log \frac{10,000}{22} &= \frac{t}{25} \log 2 \\ 25 \frac{\log \frac{10,000}{22}}{\log 2} &= t \\ 220.71 &= t \end{aligned}$$

so it takes almost 221 weeks (over 4 years) to reach 10,000 viruses.

5. Suppose a quantity  $Q$  grows exponentially at a rate of 18% per decade. Compute the doubling time of this growth.

$$T_{\text{double}} = \frac{\log 2}{\log(1.18)} = 4.1878 \text{ decades}$$

6. Suppose a quantity  $R$  decays exponentially at a rate of 44% per month. Compute the half-life of this decay.

$$T_{\text{half}} = \frac{\log .5}{\log .56} = 1.195 \text{ months}$$