Homework #5

Instructions: Answer the following questions on a **separate sheet of paper**.

1. Joanne thinks it's time to start thinking about the future. Her bank offers a savings account that compounds monthly at an APR of 3.8%. Suppose that she decides to deposit \$200 into her account each month until she retires in 35 years.

(a) What will Joanne's savings account balance be after the 35 years?

We will use the savings plan formula with PMT= \$200, $n = 12$, APR= .038 and $Y = 35$.

$$A = \$200 \left(\frac{12}{.038}\right)\left[\left(1 + \frac{.038}{12}\right)^{(12)(35)} - 1\right] = \$175,143.55.$$

(b) How much of the total was deposited by Joanne and how much did she earn in interest?

$$\text{total deposit} = (\$200)(12)(35) = \$84,000$$

$$\text{interest} = \$175,143.55 - \$84,000 = \$91,143.55$$

(c) What percentage of the balance was earned in interest?

$$\text{percentage} = \frac{\$91,143.55}{\$175,143.55} = .5204$$

so 52.04% of the total balance was earned in interest.

2. Joanne's fiercest rival Arthur hears about Joanne's plan to retire and he devises a plan to have a better retirement than her. His plan is to save enough money so that when he retires, he can live solely on the interest earned in the account. When he retires, he wants to have enough money in the bank so that he can earn a modest \$25,000 per year in interest alone. It turns out that Arthur banks at the same place as Joanne and has the same savings account that compounds monthly at an APR of 3.8%.

(a) What should the account balance be when Arthur retires so that he earns \$25,000 in interest each year?

\$25,000 should be 3.8% of the account balance, so

$$\text{account balance} = \frac{\$25,000}{.038} = \$657,894.74$$

(b) Supposing Arthur also plans to retire 35 years from now, what monthly payment does he need to make in order to reach his goal?

We will use the savings formula with A= \$657,894.74, $n = 12$, APR= .038 and $Y = 35$.

$$\$657,894.74 = \text{PMT}\ \left(\frac{12}{.038}\right)\left[\left(1 + \frac{.038}{12}\right)^{(12)(35)} - 1\right]$$

$$\$657,894.74 = \text{PMT}\ (875.7177605)$$

$$\$751.26 = \text{PMT}$$

so he should deposit \$751.26 per month.

3. Bonnie and Dexter are looking into buying a house. They secure a 30-year mortgage that compounds monthly at an APR of 7.1% for a house that costs $\$230,000$.

(a) What will Bonnie's and Dexter's monthly payments be?

We will use the loan formula with P= $230,000$, $n = 12$, APR= .071 and $Y = 30$.

$$\text{PMT} = \$230,000 \frac{\left(\frac{.071}{12}\right)}{1 - \left(1 + \frac{.071}{12}\right)^{-(12)(30)}} = \$1,545.67$$

(b) How much will they actually pay for the house?

$$\text{total paid} = (\$1,545.67)(12)(30) = \$556,441.20$$

(c) Of the total amount paid, what percentage was paid in interest?

$$\text{interest in dollars} = \$556,441.20 - \$230,000 = \$326,441.20$$

so to find the percentage of the total that is interest, we divide

$$\frac{\$326,441.20}{\$556,441.20} = .5867 = 58.67\%$$

so 58.67% of the total paid on the house went towards interest.

(d) If Bonnie and Dexter decide to instead pay off the loan in 20 years, how much will their monthly payment be and how much (in dollars) will they save in interest?

We will do the same calculations with $Y = 20$.

$$\text{PMT} = \$230,000 \frac{\left(\frac{.071}{12}\right)}{1 - \left(1 + \frac{.071}{12}\right)^{-(12)(20)}} = \$1,797.02$$

$$\text{total paid} = (\$1,797.02)(12)(20) = \$431,284.80$$

$$\text{amount saved} = \$556,441.20 - \$431,284.80 = \$125,156.40$$

4. Chester wants to buy a new car. His bank offers car loans that compound monthly at an APR of 6.2%. Suppose that he can only afford to pay \$350 per month towards a car loan.

For the following calculations, we will use PMT= \$350, $n = 12$ and APR= .062. It will be convenient to solve for P in the loan formula right from the start, yielding

$$P = \$350\left(\frac{12}{.062}\right)\left[1-\left(1+\frac{.062}{12}\right)^{-12Y}\right]$$

(a) If Chester takes out a 3-year loan, what is the highest price of car that he can afford?

$$P = \$350\left(\frac{12}{.062}\right)\left[1-\left(1+\frac{.062}{12}\right)^{-(12)(3)}\right] = \$11,470.66$$

(b) If Chester takes out a 4-year loan, what is the highest price of car that he can afford?

$$P = \$350\left(\frac{12}{.062}\right)\left[1-\left(1+\frac{.062}{12}\right)^{-(12)(4)}\right] = \$14,845.08$$

(c) If Chester takes out a 5-year loan, what is the highest price of car that he can afford?

$$P = \$350\left(\frac{12}{.062}\right)\left[1-\left(1+\frac{.062}{12}\right)^{-(12)(5)}\right] = \$18,017.15$$