Review for Exam #2

1. Linus has a savings account that compounds monthly at an APR of 5.32%.

   (a) Find the APY of Linus' account.

$$100\left(1+\frac{.0532}{12}\right)^{1}2 = 105.45$$

   so the APY is 5.45%.

   (b) If Linus deposits \$350 per month for 18 years, how much will he have after the 18 years? How much of that will have been earned in interest?

   After 18 years, Linus has a total of

$$A = 350\left(\frac{12}{0.532}\right)\left[\left(1+\frac{.0532}{12}\right)^{(12)(18)} - 1\right] = \$126,310.07$$

   The total deposited is $350(12)(18) = \$75,600$, so the amount earned in interest is

$$126,310.07 - 75,600 = \$50,710.07$$

   (c) If Linus wants to have \$125,000 10 years from now, how much should he deposit each month for the 10 years?

$$\begin{aligned}125,000 &= PMT\left(\frac{12}{0.532}\right)\left[\left(1+\frac{.0532}{12}\right)^{(12)(10)} - 1\right]\\ 125,000 &= PMT(157.9698851)\\ 791.29 &= PMT\end{aligned}$$

   So Linus needs to depost \$791.29 per month.

2. Pauline's bank offers her a loan that compounds monthly at an APR of 7.87%.

   (a) If Pauline takes out a 9-year loan for \$48,000, how much will her monthly payment be? How much will she pay over the course of the 9 years?

   The monthly payment is

$$PMT = \frac{48,000\left(\frac{.0787}{12}\right)}{1-\left(1+\frac{.0787}{12}\right)^{-(12)(9)}} = \$621.67$$

   The total amount paid over the 9 years is

$$(621.67)(12)(9) = \$67,140.36$$

(b) If Pauline can afford to pay \$800 per month for 9 years towards a loan, how much can she afford to borrow?

$$\begin{aligned} 800 &= \frac{P\left(\frac{.0787}{12}\right)}{1-\left(1+\frac{.0787}{12}\right)^{-(12)(9)}} \\ 800 &= P(.012951429) \\ 61,769.25 &= P \end{aligned}$$

So Pauline can afford a loan of \$61,769.25.

3. The number of ants on Harold's ant farm is doubling every 7 months.

(a) Find the time it takes for the number of ants to triple.
We'll use $Q = Q_0(2)^{t/T_{\text{double}}}$ with $T_{\text{double}} = 7$, $Q_0 = 1$, and $Q = 3$.

$$\begin{aligned} 3 &= 2^{t/7} \\ \log 3 &= \frac{t}{7}\log 2 \\ 7\frac{\log 3}{\log 2} &= t \\ 11.09 &= t \end{aligned}$$

So the tripling time is 11.09 months.

(b) If there were 1,250 ants two months ago, how many ants are on Harold's farm now?
Now we'll use $Q_0 = 1,250$ and $t = 2$.

$$Q = 1,250(2)^{2/7} = 1,523.77$$

so there are about 1,524 ants now.

(c) If there were 1,250 ants two months ago, how many months ago were there 200 ants? Here we'll use $Q_0 = 1,250$ again and $Q = 200$.

$$\begin{aligned} 200 &= 1,250(2)^{t/7} \\ \log\frac{200}{1,250} &= \frac{t}{7}\log 2 \\ 7\frac{\log\frac{200}{1,250}}{\log 2} &= t \\ -18.51 &= t \end{aligned}$$

There were 200 ants 18.51 months before when there were 1,250 ants, two months ago. So that means there were 200 ants 20.51 months ago.

4. The frequency of ghost sightings is decreasing at a rate of 8.2% per year.

   (a) Find the number of years it takes for the frequency of ghost sightings to be cut in half.
   We'll use $Q = Q_0(1-r)^t$ with $r = .082$, $Q_0 = 1$, and $Q = .5$.

$$\begin{aligned} .5 &= .918^t \\ \log .5 &= t\ \log .918 \\ \frac{\log .5}{\log .918} &= t \\ 8.1 &= t \end{aligned}$$

   So the frequency of ghost sightings is cut in half every 8.1 years.

   (b) If there are 88 ghost sightings per month this year, how many ghost sightings per month will there be in 11 years? How many ghost sightings per month were there 5 years ago?
   We'll use the same equation with $Q_0 = 88$ and $t = 11$ first, followed by $t = -5$.

$$Q = 88(.918)^1 1 = 34.34$$

   So 11 years from now there will be 34.34 ghost sightings per month.

$$Q = 88(.918)^{-5} = 134.98$$

   So 5 years ago there were 134.98 ghost sightings per month.

   (c) If there are 88 ghost sightings per month this year, when will there only be 2 ghost sightings per month? When were there 500 ghost sightings per month?
   We'll use the same equation as before with $Q_0 = 88$ and we'll plug in $Q = 2$ and then $Q = 500$.

$$\begin{aligned} 2 &= 88(.918)^t \\ \log \frac{2}{88} &= t\ \log .918 \\ \frac{\log \frac{2}{88}}{\log .918} &= t \\ 44.23 &= t \end{aligned}$$

   So there will be 2 ghost sightings per month 44.23 years from now.

$$\begin{aligned} 500 &= 88(.918)^t \\ \log \frac{500}{88} &= t\ \log .918 \\ \frac{\log \frac{500}{88}}{\log .918} &= t \\ -20.31 &= t \end{aligned}$$

   So there were 500 ghost sightings per month 20.31 years ago.

5. The number of fruit flies at Eleanor's compost is increasing at a rate of 2.1% per day.

   (a) How often does the number of fruit flies double? How often does the number triple?

   We'll use the equation $Q = Q_0(1+r)^t$ with $r = .021$. To find the doubling time, use $Q_0 = 1$ and $Q = 2$.

$$\begin{aligned} 2 &= (1.021)^t \\ \log 2 &= t\ \log 1.021 \\ \frac{\log 2}{\log 1.021} &= t \\ 33.35 &= t \end{aligned}$$

   So the number of fruit flies doubles every 33.35 days. To find the tripling time, we'll use $Q_0 = 1$ and $Q = 3$.

$$\begin{aligned} 3 &= (1.021)^t \\ \log 3 &= t\ \log 1.021 \\ \frac{\log 3}{\log 1.021} &= t \\ 52.86 &= t \end{aligned}$$

   So the number of fruit flies triples every 52.86 days.

   (b) If there are 465 fruit flies today, how many fruit flies were there 4 weeks ago? How many fruit flies will there be 1 year from now?

   We'll use the equation $Q = Q_0(1.021)^t$ with $Q_0 = 465$ and first $t = -28$ followed by $t = 365$.

$$Q = 465(1.021)^{-28} = 259.86$$

   So 4 weeks ago there were about 260 fruit flies.

$$Q = 465(1.021)^{365} = 915,901.59$$

   So 1 year from now there will be 915,902 fruit files.

   (c) If there are 465 fruit flies today, when will the number of fruit flies reach 2,000?

   We'll use the same equation $Q = 465(1.021)^t$ with $Q = 2000$.

$$\begin{aligned} 2000 &= 465(1.021)^t \\ \log \frac{2000}{465} &= t\ \log 1.021 \\ \frac{\log \frac{2000}{465}}{\log 1.021} &= t \\ 70.197 &= t \end{aligned}$$

   So there will be 2000 fruit flies 70.197 days from now.

6. Argon-41 has a half-life of 1.827 hours.

   (a) How long does it take for a quantity of Argon to decrease to 10% of its original amount?
   We'll use the equation $Q = Q_0(.5)^{t/T_{\text{half}}}$ where $T_{\text{half}} = 1.827$, $Q_0 = 100$, and $Q = 10$.

$$\begin{aligned} 10 &= 100(.5)^{t/1.827} \\ \log .1 &= \frac{t}{1.827} \log .5 \\ 1.827 \frac{\log .1}{\log .5} &= t \\ 6.07 &= t \end{aligned}$$

   So it will take 6.07 hours to decrease to 10% of the original 100%.

   (b) Suppose that 1 hour ago Jasper had 50mg of Argon-41. How much Argon-41 is left right now? When will Jasper only have 10mg of Argon-41?
   We'll use $Q = Q_0(.5)^{t/1.827}$ with $Q_0 = 50$. To find out how much is right now, we'll use $t = 1$.

$$Q = 50(.5)^{1/1.827} = 34.214$$

   So there are 34.214mg right now. To find when there will be 10mg left, we'll use $Q = 10$ and solve for t.

$$\begin{aligned} 10 &= 50(.5)^{t/1.827} \\ \log .2 &= \frac{t}{1.827} \log .5 \\ 1.827 \frac{\log .2}{\log .5} &= t \\ 4.242 &= t \end{aligned}$$

   So there will be 10mg left 4.242 hours from when there were 50mg, which was 1 hour ago. Therefore there will be 10mg left 3.424 hours from now.

   (c) If Jasmine has 100mg of Argon-41 right now, how long ago did she have 150mg of Argon-41. How much will she have 1 day from now?
   We'll use the formula as before, but now with $Q_0 = 100$. To find out when she had 150mg, we'll put $Q = 150$ and solve for t.

$$\begin{aligned} 150 &= 100(.5)^{t/1.827} \\ \log 1.5 &= \frac{t}{1.827} \log .5 \\ 1.827 \frac{\log 1.5}{\log .5} &= t \\ -1.07 &= t \end{aligned}$$

   So she had 150mg 1.07 hours ago. To find out how much she will have in one day, we'll put $t = 24$.

$$Q = 100(.5)^{24/1.827} = .0111$$

   So in 1 day she will only have .0111mg of Argon-41.

7. Suppose that the number of wormless apples on Exeter's apple tree is decreasing at a linear rate. Further suppose that he had 85 wormless apples 4 days after he began harvesting and 60 wormless apples 8 days after he began harvesting.

   (a) Find a linear equation that describes the number of wormless apples in Exeter's tree as a function of the number of days since he began harvesting.

   Let y be the number of wormless apples and let x be the number of days since the beginning of the harvest. Then when $x = 4$, $y = 85$ and when $x = 8$, $y = 60$. The equation of a line is $y = mx + b$ where the slope is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{60 - 85}{8 - 4} = \frac{-25}{4} = -6.25$$

   Therefore the equation is $y = -6.25x + b$. To find b, we just plug in either of the two points, say $x = 4$, $y = 85$.

$$\begin{aligned} 85 &= -6.25(4) + b \\ 85 &= -25 + b \\ 110 &= b \end{aligned}$$

   So the desired equation is

$$y = -6.25x + 110$$

   (b) How many wormless apples did he have the day he began harvesting? How many wormless apples did he have 15 days after he began harvesting?

   The number of wormless apples the day harvesting began is the y-value corresponding to $x = 0$.

$$y = -6.25(0) + 110 = 110$$

   So Exeter had 110 wormless apples when he began harvesting them. To find the number of wormless apples 15 days later, we plug in $x = 15$.

$$y = -6.25(15) + 110 = 16.25$$

   So after 15 days, there are only about 16 wormless apples left.

   (c) How many days after he began harvesting apples were there 30 wormless apples?

   To find when there were 30 wormless apples, we put $y = 30$ and solve for x.

$$\begin{aligned} 30 &= -6.25x + 110 \\ -80 &= -6.25x \\ 12.8 &= x \end{aligned}$$

   So there were 30 wormless apples 12.8 days after he began harvesting them.

8. The price of robot vacuums is increasing by \$4.10 per month and on average they already cost \$280!

   (a) Find a linear equation that describes the price or robot vacuums as a function of the number of months from right now.

   Let y be the price of robot vacuums and let x be the number of months from now. The equation of a line is $y = mx + b$ where the slope m is given by the change in y over the change in x. But the cost changes by 4.10 every 1 month, so the slope must be

   $$m = 4.1$$

   So far, then, the equation is $y = 4.1x + b$. Since b is the y-value corresponding to $x = 0$, then b must be the cost right now, which is 280. Hence $b = 280$ and the equation is

   $$y = 4.1x + 280$$

   (b) How much will robot vacuums cost one year from now?

   One year from now is $x = 12$.

   $$y = 4.1(12) + 280 = 329.2$$

   So one year from now they will cost \$329.20.

   (c) When will robot vacuums cost \$1,000?

   To find when they will cost \$1000, we set $y = 1000$ and solve for x.

   $$\begin{aligned} 1000 &= 4.1x + 280 \\ 720 &= 4.1x \\ 175.61 &= x \end{aligned}$$

   So they will cost \$1000 after 175.61 months; that is, 14.63 years from now.

9. A model version of a T-Rex is 2 1/2 feet tall with a surface area of 3 square feet and a volume of 1 cubic foot. If the actual T-Rex (which is the proportionally identical to the model) is 18 feet tall, what is its volume and surface area?

   The scaling factor is

   $$SF = \frac{\text{new height}}{\text{old height}} = \frac{18 \text{ ft}}{2.5 \text{ ft}} = 7.2$$

   Therefore, the surface area and volume of the actual T-Rex are scaled as follows:

   $$\text{new surface area} = (\text{old surface area})(SF)^2 = (3 \text{ ft}^2)(7.2)^2 = 155.52 \text{ ft}^2$$

   $$\text{new volume} = (\text{old volume})(SF)^3 = (1 \text{ ft}^3)(7.2)^3 = 373.248 \text{ ft}^3$$

10. Compute the following:

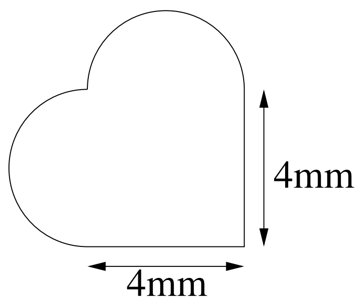
(a) The volume of a cylindrical tank that is 100 feet tall and 150 feet in diameter.
If the diameter of a tank is 150 feet, then the radius is 75 feet. The volume of a cylinder is

$$V = \pi r^2 h = \pi(75\text{ ft})^2(150\text{ ft}) = 1,767,145.868\text{ ft}^3$$

(b) The volume of a sphere of radius 8 yards.
The volume of a sphere is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(8\text{ yd})^3 = 2,144.66\text{ yd}^3$$

(c) The perimeter and area of the figure below:



The perimeter of the figure is the combined length of the two straight pieces added to the combined length of the two semicircles. Since the length of two semicircles of the same size is the same as the perimeter of a circle of radius 2 mm, then the total perimeter is

$$\text{perimeter} = 2\pi(2\text{ mm}) + 4\text{ mm} + 4\text{ mm} = (4\pi + 8)\text{ mm} = 20.566\text{ mm}$$

The area of the figure is the area of the two semicircles plus the area of the square. Since the two semicircles together make one circle of radius 2 mm and since the square and side length 4mm, then the total area is

$$\text{area} = \pi(2\text{ mm})^2 + (4\text{ mm})(4\text{ mm}) = (4\pi + 16)\text{ mm}^2 = 28.566\text{ mm}^2$$