Homework #8

Instructions: Do the following problems on a **separate** sheet of paper.

1. Solve for x in the following equations.

(a) $\log_x 81 = 4$

This means that $x^4 = 81$, so $x = 3$ since $3^4 = 81$.

(b) $\log_7 x = \dfrac{1}{2}$

This means that $7^{1/2} = x$, i.e., $x = \sqrt{7}$.

(c) $\log_{125} 5 = x$

This means that $125^x = 5$. Since $5^3 = 125$, then we have $(5^3)^x = 5$; that is, $5^{3x} = 5^1$. So $x = \frac{1}{3}$.

(d)

$$\begin{aligned} 1.15^x &= 80 \\ \log 1.15^x &= \log 80 \\ x \log 1.15 &= \log 80 \\ x &= \frac{\log 1.15}{\log 80} \\ x &= 31.35 \end{aligned}$$

(e)

$$\begin{aligned} 400 &= 650(.5)^{x/10} \\ \frac{400}{650} &= .5^{x/10} \\ \log \frac{400}{650} &= \log \left(.5^{x/10}\right) \\ \log \frac{400}{650} &= \frac{x}{10} \log .5 \\ 10 \frac{\log \frac{400}{650}}{\log .5} &= x \\ 7.004 &= x \end{aligned}$$

2. Suppose that Pollyanna has a viral infection that is being treated by an herbal remedy, and that the number of viruses is decreasing by 8.2% per day.

For this problem we will use $1 - r$ since it's decreasing with $r = .082$.

$$Q = Q_0(1 - .082)^t = Q_0(.918)^t$$

(a) What percentage of the original viruses are left after 2 weeks?

Since we are talking about the percentage of viruses, the initial percentage is 100%. So we will use $Q_0 = 100$. Since r is given per day, we must compute t in days. 2 weeks is 14 days, so $t = 14$.

$$Q = 100(.918)^14 = 30.185$$

so after 2 weeks there is 30.185% of the original viruses left.

(b) After how many days will the number of viruses be at 16% of the original amount?

Now we will use $Q = 16$ with $Q_0 = 100$ to solve for t.

$$\begin{aligned} 16 &= 100(.918)^t \\ .16 &= .918^t \\ \log .16 &= \log .918^t \\ \log .16 &= t \log .918 \\ \frac{\log .16}{\log .918} &= t \\ 21.42 &= t \end{aligned}$$

So it takes 21.42 days to decrease to 16% of the original amount.

(c) How often is the number of viruses cut in half?

Now we will use $Q = 50$ and $Q_=100$ (or $Q_0 = 1$, $Q = .5$).

$$\begin{aligned} 50 &= 100(.918)^t \\ .5 &= .918^t \\ \log .5 &= t \ \log .918 \\ 8.101 &= t \end{aligned}$$

So the number of viruses is cut in half every 8.101 days.

3. Suppose that Rulon has had a viral infection for the last 2 months that just won't go away no matter what he tries, and the the number of viruses is increasing by 13.55% per week. Further suppose that Rulon's infection consisted of a modest 22 viruses 2 weeks ago.

For this problem we will use $1 + r$ since it's increasing with $r = .1355$. Also, $Q_0 = 22$ and we will count time starting 2 weeks ago. Our equation is

$$Q = 22(1.1355)^t$$

where t is counted in weeks.

(a) How many viruses does he have right now?

Right now is 2 weeks past the starting point, so $t = 2$.

$$Q = 22(1.1355)^2 = 28.3659$$

So Rulon has 28 viruses right now.

(b) How many viruses will he have tomorrow?

Tomorrow is 15 days past the starting point, which is $2 + \frac{1}{7}$ weeks past the starting point.

$$Q = 22(1.1355)^{2+\frac{1}{7}} = 28.8856$$

So Rulon will have about 29 viruses tomorrow.

(c) How many viruses did he have 7 weeks ago?

7 weeks ago is 5 weeks before the starting point, so $t = -5$.

$$Q = 22(1.1355)^{-5} = 11.6543$$

So Rulon had 12 viruses 7 weeks ago.

(d) How often does the number of viruses double?

To find the doubling time, we could pick double the starting value for Q. So $Q = 44$. (or we could use $Q_0 = 1$, $Q = 2$.

$$\begin{aligned} 44 &= 22(1.1355)^t \\ 2 &= 1.1355^t \\ \log 2 &= t \ \log 1.1355 \\ \frac{\log 2}{\log 1.1355} &= t \\ 5.4547 &= t \end{aligned}$$

So the number of viruses doubles every 5.45 weeks.