

Homework #6

Instructions: Do the following problems on a **separate** sheet of paper.

1. Joanne believes that it's time to start thinking about her future. Her bank offers a savings account that compounds monthly at an APR of 3.8%. Suppose that she decides to deposit \$200 into her account each month until she retires in 35 years.

(a) What will Joanne's savings account balance be after the 35 years?

$$A = 200 \left(\frac{12}{.038} \right) \left[\left(1 + \frac{.038}{12} \right)^{(12)(35)} - 1 \right] = 175,143.55$$

so she will have \$175,143.55 after 35 years.

(b) How much of the total was deposited by Joanne and how much did she earn in interest.

$$\begin{aligned} \text{deposited} &= (200)(12)(35) = 84,000 \\ \text{interest} &= 175,143.55 - 84,000 = 91,143.55 \end{aligned}$$

(c) What percentage of the balance was earned in interest?

The desired percentage will be obtained by dividing the interest by the balance.

$$\frac{91,143.55}{175,143.55} = .5203$$

so 52.03% of the balance came from interest.

2. Joanne's fiercest rival Arthur hears about Joanne's plan to retire and he devises a plan to have a better retirement than her. His plan is to save enough money so that when he retires, he can live solely on the interest earned in the account. When he retires, he wants to have enough money in the bank so that he can earn a modest \$25,000 per year in interest alone. It turns out that Arthur banks at the same place as Joanne and has the same savings account that compounds monthly at an APR of 3.8%.

(a) What should the account balance be when Arthur retires so that he earns \$25,000 in interest each year?

First, we find the APY.

$$100 \left(1 + \frac{.038}{12} \right)^{12} = 103.867$$

so the account will grow by 3.867% each year. So \$25,000 should be 3.867% of the needed balance; that is, $\$25,000 = .03867x$ where x is the balance.

$$x = \frac{25,000}{.03867} = 646,495.99$$

so he needs to have a total of \$646,495.99 in his account when he retires.

- (b) Supposing Arthur also plans to retire 35 years from now, what monthly payment does he need to make in order to reach his goal?

$$646,495.99 = \text{PMT} \left(\frac{12}{.038} \right) \left[\left(1 + \frac{.038}{12} \right)^{(12)(35)} - 1 \right]$$

$$646,495.99 = \text{PMT}(875.7177639)$$

$$738.25 = \text{PMT}$$

so he needs to make monthly deposits of \$738.25.

3. Bonnie and dexter are looking into buying a house. They secure a 30-year mortgage that compounds monthly at an APR of 7.1% for a house that costs \$230,000.

- (a) What will Bonnie's and Dexter's monthly payments be?

$$\text{PMT} = \frac{230,000 \left(\frac{.071}{12} \right)}{1 - \left(1 + \frac{.071}{12} \right)^{-(12)(30)}} = \frac{230,000 \left(\frac{.071}{12} \right)}{.8804144776} = 1,545.67$$

so the monthly payment will be \$1,545.67.

- (b) How much will they actually pay for the house?

$$\text{total paid} = 1,545.67(12)(30) = 556,441.20$$

- (c) Of the total amount paid, what percentage was paid in interest? The desired percentage is gotten by dividing interest by total paid. Since interest is the cost of the house subtracted from the total paid,

$$\frac{556,441.20 - 230,000}{556,441.20} = .5866$$

so 58.66% of the total paid went toward interest.

- (d) If Bonnie and Dexter decide to instead pay off the loan in 20 years, how much will their monthly payment be and how much (in dollars) will they save in interest?

$$\text{PMT} = \frac{230,000 \left(\frac{.071}{12} \right)}{1 - \left(1 + \frac{.071}{12} \right)^{-(12)(20)}} = \frac{230,000 \left(\frac{.071}{12} \right)}{.7572724513} = 1,797.02$$

so their monthly payment would be \$1,797.02, meaning that over the 20 years they would pay a total of

$$\text{total paid} = 1,797.02(12)(20) = 431,284.80$$

towards the house. This would save them

$$556,441.20 - 431,284.80 = 124,156.40$$

that is, they would save \$124,156.40 in interest by paying it off in 20 years.

4. Chester wants to buy a new car. His bank offers car loans that compound monthly at an APR of 6.2%. Suppose that he can only afford to pay \$350 per month towards a car loan.

(a) If Chester takes out a 3-year loan, what is the largest loan amount that he can afford?

$$350 = \frac{P \left(\frac{.062}{12} \right)}{1 - \left(1 + \frac{.062}{12} \right)^{-(12)(3)}}$$

If we solve for P we get

$$P = 350 \left[1 - \left(1 + \frac{.062}{12} \right)^{-(12)(3)} \right] \left(\frac{12}{.062} \right) = 11,470.66$$

so he can afford an \$11,470.66 car.

(b) If Chester takes out a 4-year loan, what is the largest loan amount that he can afford?

$$P = 350 \left[1 - \left(1 + \frac{.062}{12} \right)^{-(12)(4)} \right] \left(\frac{12}{.062} \right) = 14,845.08$$

(c) If Chester takes out a 5-year loan, what is the largest loan amount that he can afford?

$$P = 350 \left[1 - \left(1 + \frac{.062}{12} \right)^{-(12)(5)} \right] \left(\frac{12}{.062} \right) = 18,017.15$$