

Homework #5 Solutions

Fereidoun M. Esfandiary, a futurist and transhumanist, was born in Iran in 1930. He lived in 17 countries by the time he was 11. In 1970, he changed his name to FM-2030 because “conventional names define a person’s past: ancestry, ethnicity, nationality, religion. [. . .] The name 2030 reflects my conviction that the years around 2030 will be a magical time.”

1. With the sales from FM-2030’s first non-fiction book *Upwingers: A Futurist Manifesto* in 1973, he decided to open a savings account. Being a transhumanist, FM wanted to get the most for his money. His bank offered an account with an APR of 7.43% compounded quarterly or an account with an APR of 7.37% compounded continuously.

(a) Compute the APY for both of the accounts to determine which account is the better investment.

To calculate the APY we use $P = 100$ and $Y = 1$. The first account gives

$$A = 100 \left(1 + \frac{.0743}{4} \right)^4 = 107.63959$$

making the APY of the first account 7.63959%. For the second account

$$A = 100e^{.0737} = 107.64838$$

the APY is 7.64838%. So the second account is the better investment.

(b) If FM-2030 deposited \$15,000 into the better account, how much did he have in the account 27 years later in the year 2000?

We’ll use the second account that compounds continuously at an APR of 7.37% with $P = 15,000$ and $Y = 27$.

$$A = 15,000e^{(.0737)(27)} = 109,722.03$$

so in the year 2000, FM will have \$109,722.03 in his account.

(c) Of the total amount in the year 2000, what percentage was deposited and what percentage was earned in interest?

The question translates as: 15,000 is what percent of 109,722.03? Let x be the decimal version of the desired percent. Then $15,000 = x(109,722.03)$, making

$$x = \frac{15,000}{109,722.03} = .1367$$

so of the total amount in the year 2000 13.67% was deposited and the remaining 86.33% was earned in interest.

2. As a futurist, FM-2030 decided that cryopreservation was for him. The Alcor Life Extension Foundation in Scottsdale, Arizona, charges \$150,000 to cryogenically freeze an entire body.

- (a) If he wanted to have the \$150,000 by the year 1995 and his savings account compounded continuously at an APR of 5.8%, how much did he need to deposit in 1975 in order to reach his goal?

We'll use $APR = .058$, $Y = 20$, and $A = 150,000$ in the continuous interest formula to solve for the deposited amount P .

$$\begin{aligned}150,000 &= Pe^{(.058)(20)} \\150,000 &= P(3.189933276) \\ \frac{150,000}{3.189933276} &= P \\47,022.93 &= P\end{aligned}$$

So he needed to deposit \$47,022.93 in 1975.

- (b) If he wanted to have the \$150,000 by the year 2000 and his savings account compounded monthly at an APR of 4.8%, how much did he need to deposit in 1975 in order to reach his goal?

This time we'll use $APR = .048$, $Y = 25$, $n = 12$, and $A = 150,000$ in the monthly compounding formula to solve for P .

$$\begin{aligned}150,000 &= P \left(1 + \frac{.048}{12} \right)^{(12)(25)} \\150,000 &= P(3.312179331) \\45,287.40 &= P\end{aligned}$$

So he needed to deposit \$45,287.40 in 1975.

3. In addition to the \$150,000 freezing fee, there is also a yearly membership fee of \$500. FM-2030 knew that he might be in cryopreservation for hundreds of years. So he needed a savings account that earns \$500 in interest every year.

- (a) If his bank account has an APR of 5.5%, how much money does he need in the account in order to generate the \$500 yearly amount?

Since the APY is close to the APR, we can use the APR as an estimate. So we want to find out what 500 is 5.5% of. Let x be the needed amount. Then $500 = .055x$, giving

$$x = \frac{500}{.055} = 9090.91$$

So if FM has \$9,090.91 in his account, the account will generate the needed yearly fee of \$500.

- (b) Suppose that his account with the APR of 5.5% compounds daily. If he decided to start the account in 1980, how much should he have deposited in order to have the amount calculated in part (a) by the year 2000?

We'll use $APR = .055$, $Y = 20$, $n = 365$, and $A = 9,090.91$ in the daily compounding formula to solve for P .

$$\begin{aligned}9,090.91 &= P \left(1 + \frac{.055}{365} \right)^{(365)(20)} \\9,090.91 &= P(3.003917051) \\3,026.35 &= P\end{aligned}$$

So he needs to deposit \$3,026.35 in his account in 1980 so that in 2000 his account will generate enough interest to cover his yearly \$500 fee.

4. Unfortunately, FM-2030 did not make it to year 2030 alive. He died of pancreatic cancer on July 8th, 2000. However, he was cryogenically frozen as planned. Suppose he took his remaining \$100,000 and deposited it in a bank account that compounds every 2 months at an APR of 6%.

- (a) If FM-2030 is unfrozen in the year 2415, how much will he have in his bank account?

We'll use $APR = .06$, $Y = 415$, $n = 6$, and $P = 100,000$ in the compound interest formula.

$$A = 100,000 \left(1 + \frac{.06}{6} \right)^{(6)(415)} = 5,757,324,638,000,000$$

so he'll have about 5.7 quadrillion dollars in the year 2415.

- (b) What percent more than the deposited \$100,000 is the account total in the year 2415?

The question translates as: 5,757,324,638,000,000 is what percent more than 100,000, i.e., $5,757,324,638,000,000 = x(100,000)$, which has solution

$$x = 57,573,246,380$$

which converted to a percent gives that 5,757,324,638,000,000 is 5,757,324,638,000% of 100,000, meaning that it is 5,757,324,637,900% more than 100,000. So his account increased by over 5.7 trillion percent!!