In each of the following questions you will be asked to find the integral (or integrals) that compute something (lengths, areas, volumes or centroids). You just need to write down the definite integral that would give the solution: there is NO NEED TO COMPUTE ANY INTEGRAL in this test. The value of every question is indicated at the beginning of it. You may only use scratch paper and a small note card. No cell phones, calculators, notes, books or music players are allowed during the test.

Name: _____ UID: _____

1. (15 points)

   (i) (5 points) Let R be the region bounded by the graphs of $x = 3 - y^2$, $y = x - 1$ and $y = 0$. Write down the integral that would compute the area of R using vertical slices.

   (ii) (5 points) Write down the integral that would compute the area of R using horizontal slices.

   (iii) (5 points) Now consider the region bounded by the graphs of $x = 3 - y^2$, $y = x - 1$ and $x = 0$. Write down the integral that would compute the area of R.

2. (15 points) Let R be the region in the first quadrant bounded by the graphs of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $y=0$.

   (i) Write down the integral that would compute the volume of the solid obtained by revolving R about the x-axis.

   (a) (5 points) Using shells:

   (b) (5 points) Using disks:

   (ii) (5 points) Use any method to write down the integral that would compute the volume of the solid obtained by revolving R about the line $y=-1$.

3. (10 points) Let R be the region bounded by the graphs of $y = 4x$ and $y = 4x^2$. Write down the integral that would compute the volume of the solid obtained by revolving R about the y-axis.

   (i) (5 points) Using shells:

   (ii) (5 points) Using disks:

4. (15 points) For each of the following curves, write down the integral that would compute its arc length.

   (i) (5 points) $y = x^{3/2}, \quad 1 \leq x \leq 4.$

   (ii) (5 points) $30xy^3 - y^8 = 15, \quad y = 1, \quad y = 3.$

   (iii) (5 points) $x = 3t^2 + 2, \quad y = 2t^3 - \frac{1}{2}, \quad 1 \leq t \leq 4.$

5. (10 points) Write down the integral that would compute the area of the surface generated by revolving the following curves about the x-axis.

   (i) (5 points) $y = \frac{x^6+2}{8x^2}, \quad 1 \leq x \leq 3.$

   (ii) (5 points) $x = t, \quad y = t^3, \quad 0 \leq t \leq 1$ (sketch a graph of this curve).

6. EXTRA CREDIT (10 points) The integral

$$\pi \int_a^b [f(x)]^2 \, dx$$

computes the volume of the solid obtained by revolving the region under the curve $y = f(x), \quad a \leq x \leq b$ about the x-axis.

Use a picture and a couple of sentences to justify why this is the case.