Section A: Conceptual questions.

1. (5 points) If $f$ is a function which is differentiable over the open interval $(a, b)$ and continuous over the closed interval $[a, b]$, the mean value theorem for derivatives states that there exists some $c$ inside of $[a, b]$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Consider the function $f(x) = x^{2/3}$ over the interval $[-8, 27]$. Show that the mean value theorem for derivatives fails and explain why.

2. (5 points) Recall that a function $f$ is odd if $f(-x) = -f(x)$ for all $x$. Show that if $f$ is an odd function, then for any real number $a$ we have

$$\int_{-a}^{a} f(x) \, dx = 0$$

*Hint: use the change of variables $u = -x$.**
Section B: Practical questions.

3. (15 points) Compute the following indefinite integrals:
   
   (i) (5 points) \( \int \frac{(x^2+1)^2}{\sqrt{x}} \, dx \)

   (ii) (5 points) \( \int \frac{3x}{\sqrt{2x^2+5}} \, dx \)

   (iii) (5 points) \( \int x^2(x^3 + 5)^8 \cos [(x^3 + 5)^9] \, dx \)

   *Hint: Use the change of variables \( u = (x^3 + 5)^9 \).*
4. (15 points) Find the area under the curve \( y = 2x + 2 \) over the interval \([-1, 1]\) as follows.

(i) (2 points) Subdivide the interval \([-1, 1]\) into \(n\) equal subintervals

\[ [x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n] \]

What is the length \( \Delta x \) of every subinterval? For every \( k \), write an expression for \( x_k \).

\[ \Delta x = \quad x_k = \]

(ii) (4 points) Write down an expression for the area of the rectangle over \([x_k, x_{k+1}]\) which depends ONLY on \( k \) and \( n \).

(iii) (7 points) Find the sum \( A(R_n) \) of the areas of the \( n \) rectangles.

*Hint: Remember that \( \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \).*

(iv) (2 points) Find the limit \( A = \lim_{n \to \infty} A(R_n) \).
5. (15 points) Consider the function \( F(x) = \int_{\sin x}^{\cos x} t^5 \, dt \)

(i) (2 points) Compute \( F \left( \frac{\pi}{4} \right) \) and \( F(0) \).

(ii) (10 points) Compute \( F'(x) \)

\[ \text{Hint: Write } F(x) = \int_{\sin x}^{\cos x} t^3 \, dt + \int_{\cos x}^{\cos x} t^3 \, dt. \]

6. (10 points) Consider the function \( f(x) = \int_{0}^{x} \frac{u}{\sqrt{1+u^2}} \, du \)

(i) (5 points) Find the intervals where \( f \) is increasing or decreasing.

(ii) (5 points) Find the intervals where \( f \) is concave up or down.
7. (10 points) Compute the definite integral
\[
\int_{0}^{\pi/2} \sin x \sin(\cos x) \, dx
\]

8. (10 points) Use symmetry to compute the following integral
\[
\int_{-\pi/2}^{\pi/2} \frac{\sin x}{1 + \cos x} + |x| \sin^5 + x^2 \, dx
\]

Be explicit in your justification.
9. (15 points) YOU DON’T HAVE TO COMPUTE ANY INTEGRAL IN THIS EXERCISE.

(i) (5 points) Let $R$ be the region bounded by the graphs of $y = \sqrt{x}$, $y = -x + 6$ and $y = 0$. Write down the integral that would compute the area of $R$ using vertical slices.

(ii) (5 points) Write down the integral that would compute the area of $R$ using horizontal slices.

(iii) (5 points) Now consider the region bounded by the graphs of $y = \sqrt{x}$, $y = -x + 6$ and $x = 0$. Write down the integral that would compute the area of $R$. 

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