This exam consists of 2 sections, A and B. Section A is conceptual, whereas section B is more computational. The value of every question is indicated at the beginning of it. You may only use scratch paper and a small note card. No cell phones, calculators, notes, books or music players are allowed during the test.

Name: ______________________________ UID: ______________

Section A: Conceptual questions.

1. (5 points) If f is a function which is differentiable over the open interval (a, b) and continuous over the closed interval $[a, b]$, the mean value theorem for derivatives states that there exists some c inside of $[a, b]$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

   Consider the function $f(x) = x^{2/3}$ over the interval $[-8, 27]$. Show that the mean value theorem for derivatives fails and explain why.

2. (5 points) Recall that a function f is odd if $f(-x) = -f(x)$ for all x. Show that if f is an odd function, then for any real number a we have

$$\int_{-a}^{a} f(x)\, dx = 0$$

   Hint: use the change of variables $u = -x$.

Section B: Practical questions.

3. (15 points) Compute the following indefinite integrals:

   (i) (5 points) $\int \frac{(x^2+1)^2}{\sqrt{x}}\, dx$

   (ii) (5 points) $\int \frac{3x}{\sqrt{2x^2+5}}\, dx$

   (iii) (5 points) $\int x^2(x^3+5)^8 \cos\left[(x^3+5)^9\right]\, dx$
   Hint: Use the change of variables $u = (x^3+5)^9$.

4. (15 points) Find the area under the curve $y = 2x + 2$ over the interval $[-1, 1]$ as follows.

(i) (2 points) Subdivide the interval $[-1, 1]$ into n equal subintervals

$$[x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n]$$

What is the length Δx of every subinterval? For every k, write an expression for x_k.

$$\begin{aligned} \Delta x &= \\ x_k &= \end{aligned}$$

(ii) (4 points) Write down an expression for the area of the rectangle over $[x_k, x_{k+1}]$ which depends ONLY on k and n.

(iii) (7 points) Find the sum $A(R_n)$ of the areas of the n rectangles.
Hint: Remember that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

(iv) (2 points) Find the limit $A = \lim_{n\to\infty} A(R_n)$.

5. (15 points) Consider the function $F(x) = \int_{\sin x}^{\cos x} t^5 \, dt$

   (i) (2 points) Compute $F\left(\frac{\pi}{4}\right)$ and $F(0)$.

   (ii) (10 points) Compute $F'(x)$
   Hint: Write $F(x) = \int_{\sin x}^{a} t^3 \, dt + \int_{a}^{\cos x} t^3 \, dt$.

6. (10 points) Consider the function $f(x) = \int_0^x \frac{u}{\sqrt{1+u^2}} \, du$

   (i) (5 points) Find the intervals where f is increasing or decreasing.

   (ii) (5 points) Find the intervals where f is concave up or down.

7. (10 points) Compute the definite integral

$$\int_0^{\pi/2} \sin x \sin(\cos x)\, dx$$

8. (10 points) Use symmetry to compute the following integral

$$\int_{-\pi/2}^{\pi/2} \frac{\sin x}{1+\cos x} + |x| \sin^5 + x^2\, dx$$

Be explicit in your justification.

9. (15 points) YOU DON'T HAVE TO COMPUTE ANY INTEGRAL IN THIS EXERCISE.

   (i) (5 points) Let R be the region bounded by the graphs of $y = \sqrt{x}$, $y = -x + 6$ and $y = 0$. Write down the integral that would compute the area of R using vertical slices.

   (ii) (5 points) Write down the integral that would compute the area of R using horizontal slices.

   (iii) (5 points) Now consider the region bounded by the graphs of $y = \sqrt{x}$, $y = -x + 6$ and $x = 0$. Write down the integral that would compute the area of R.