Math 1210 Midterm 1 January 31st, 2014

This exam consists of 2 sections, A and B. Section A is conceptual, whereas section B is more computational. The value of every question is indicated at the beginning of it. You may only use scratch paper and a small note card. No cell phones, calculators, notes, books or music players are allowed during the exam.

Name: _____ UID: _____

Section A: Conceptual questions.

1. (i) (4 points) What does it mean for a function $f(x)$ to be continuous at a point $x = c$?

   (ii) (4 points) What is the derivative of a function $f(x)$ at $x = c$ (write it at as a limit)?

   (iii) (4 points) Give a geometric interpretation of the derivative of $f(x)$ in terms of the curve $y = f(x)$.

2. (8 points) In the Figure 1 below you can see the graph of a function $f(x)$ and the graph of its derivative $f'(x)$. Indicate **which one is which** and support your answer by giving **at least two reasons**.

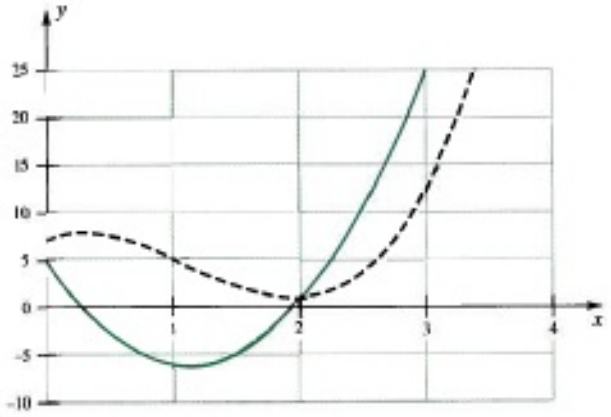

Figure 1: Question 2

3. (4 points) Use the definition (limit) to compute the derivative of the function

$$f(x) = 2x^3 + 5$$

Recall that $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

4. (8 points) Find the equation of the tangent line to the curve $y = \tan x$ at the point $\left(\frac{\pi}{4}, 1\right)$

Section B: Practical questions.

5. (16 points) Study the continuity of the function

$$f(x) = \begin{cases} \frac{3}{x-2}, & x < -1 \\ x, & -1 \leq x \leq 1 \\ \frac{-x^2+4x-3}{x-3}, & x > 1 \end{cases}$$

by answering the following questions.

(i) (2 points) What is the domain of the function f (namely, at what points is f defined)?

(ii) (6 points) Are there any removable discontinuities? If so, what is the limit at those points? (namely, if there is a removable discontinuity at $x = c$, compute $\lim_{x\to c} f(x)$).

(iii) (6 points) Are there any jump discontinuities? If so, what are the right-hand and left-hand limits?

(iv) (2 points) Are there any vertical asymptotes? If so, what are the right-hand and left-hand limits ($-\infty$ or $+\infty$)?

6. (4 points) Use the intermediate value theorem to show that the equation

$$x^3 - 7x^2 + 14x - 8 = 0$$

has a solution in the interval [0,5] (Recall that a solution of the equation $f(x) = 0$ is a number c such that $f(c) = 0$.)

7. (4 points) Compute the limit

$$\lim_{x\to\infty} \sqrt[3]{\frac{1+8x^2}{x^2+4}}$$

8. (4 points) Compute the limit

$$\lim_{x\to 0} \frac{3x\tan x}{\sin x}$$

9. (4 points) Compute the limit

$$\lim_{x\to\sqrt{2}} \frac{\sqrt[3]{2x^2+3}}{7x^2+13}$$

10. (4 points) Compute the limit

$$\lim_{x\to -2} \frac{(x+2)(x^2-x-6)}{x^2+4x+4}$$

11. (4 points) Compute the limits

$$\lim_{x\to 2^+} \frac{x+1}{x^2-5x+6}, \qquad \lim_{x\to 2^-} \frac{x+1}{x^2-5x+6}$$

12. (20 points) Compute the derivatives of the following functions (you don't need to simplify your solution).

(i) (4 points) $f(x) = (x^4 + 1)(x^2 + 1)$

(ii) (4 points) $f(x) = \frac{5x^2+2x-6}{3x-1}$

(iii) (4 points) $f(x) = \sin x \tan x$

(iv) (8 points) $f(x) = \frac{x \cos x + \sin x}{x^2+1}$