

MATH 1210-6
Spring 2003
Midterm exam III

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| Student Name: _____ |
| Student ID Number: _____ |

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| Course Abbreviation and Number: | <i>Math 1210</i> |
| Course Title: | <i>Calculus I</i> |
| Instructor: | <i>Vladimir Vinogradov</i> |

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| Date of Exam: | <i>April 7, 2003</i> |
| Time Period: | Start time: <i>12:55 pm</i> End Time: <i>1:55 pm</i> |
| Duration of Exam: | <i>1 hours</i> |
| Number of Exam Pages: | <i>6</i> |
| (including this cover sheet) | |
| Exam Type: | <i>Closed Book</i> |
| Additional Materials Allowed: | <i>Calculator</i> |

| QUESTION | VALUE | SCORE |
|--------------|------------|-------|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |
| TOTAL | 100 | |

*) The bonus question counts for 10 points maximum.

1. (20 points) Find the indefinite integral of:

$$\int 24x(x^2 - 1)^3 dx$$

ANSWER: _____

2. (20 points) Find the function whose value at 1 is 0 and whose derivative is given:

$$\frac{dy}{dx} = \frac{3(x^2 + 1)}{(x^3 + 3x)^2}$$

ANSWER: _____

3. (20 points) Calculate the definite integrals:

$$\int_{\pi/6}^{\pi/3} 4(\sin(4x) - \cos(4x)) dx$$

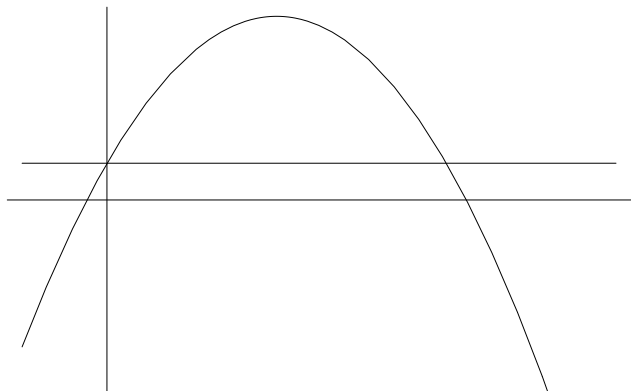
ANSWER: _____

4. (20 points) Find the solution to the following differential equation such that $y(0) = 1$

$$\frac{dy}{dx} = \sqrt{x}y^2 + 2x^2y^2$$

ANSWER: _____

5. (20 points) What is the area of the region bounded by the curves $y_1 = 5 - (x - 2)^2$ and $y_2 = 1$.



ANSWER: _____

Bonus question (10 points). Evaluate

$$\frac{d}{dx} \int_{-x^2}^{x^2} f(t) dt$$

ANSWER: _____

Useful formulae

Differentiation

Product rule: $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$

Quotient rule: $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$

Chain rule: $\frac{d}{dx}f(g(x)) = \frac{df}{dg} \cdot \frac{dg}{dx}$

Power rule $(x^\alpha)' = \alpha x^{\alpha-1}$

Trigonometric functions: $(\cos x)' = -\sin x$, $(\sin x)' = \cos x$

Integration

$$\int f(g(x))g'(x)dx = \int f(u)du = F(g(x)) + C$$
$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du = F(g(b)) - F(g(a))$$

$$\cos(0) = 1, \sin(0) = 0$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}, \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}, \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{2}\right) = 0, \sin\left(\frac{\pi}{2}\right) = 1$$

$$\cos(\pi) = -1, \sin(\pi) = 0$$