

Example

1. Let $y = (x^2 - 1)(x^2 - 5)$. For what value of x in the interval $[-2, 2]$ is y a maximum? a minimum? Find the points of inflection of the graph.

2. Let

$$y = 10 + 7x + 2x^2 - x^3.$$

For what value of x is y a local maximum? a local minimum?

3. For the following function, find a) all critical values, b) intervals in which the function is increasing and where it is decreasing, c) intervals in which the function is concave up or concave down:

$$y = x^4 - 4x^3 + 4x^2 + 2$$

Sketch the graph.

4. Consider the function $y = \sin x + (\cos x)^2$ as defined on the interval $[-\pi, \pi]$. Find a) all critical values, b) all points of inflection, c) the value at which the function takes its maximum. Sketch the graph.

5. Let

$$y = \frac{1}{x^2 - 1}.$$

Find the x coordinate of the points of inflection of the curve so defined. Sketch the graph.

6. Graph

$$y = \frac{x(x - 2)}{x^2 - 1} = \frac{x^2 - 2x}{x^2 - 1}$$

You must show enough work to explain how you found the various features of the graph.

7. In each of the following, derive enough information about the curve to sketch the graph, showing clearly all asymptotes.

$$y = \frac{x}{x^2 + 1}$$

$$y = \frac{x}{(x - 1)(x - 2)}$$

$$y = \frac{x^2}{(x - 1)(x - 2)}$$

$$y = \sqrt{1 - \cos x}$$

8. I have to make a closed cylindrical can to hold 12 cu. ft. The material to make the top and bottom costs \$6 a sq. ft., and the material to make the side costs \$10 per sq. ft. What are the dimensions which minimize the cost? (It will suffice to give either the radius of the base or the height).

9. I want to build a rectangular building on a square base which has 60000 ft³ of space. My painter will charge $\$h/2$ per ft² to paint a building h feet high, and will paint the roof at a cost of $\$30$ per ft². How high should the building be to minimize the cost of painting?
10. I have an elliptical meadow whose major axis is 100 meters and whose minor axis is 60 meters. I want to put a rectangular soccer field in the meadow, two of whose sides are parallel to the major axis, What are the dimensions of the field which maximize its area?

