MATH 1220-6
Fall 2003
Midterm exam I

Student Name: ______________________________

Student ID Number: ______________

Course Abbreviation and Number:	*Math 1220*
Course Title:	*Calculus II*
Instructor:	*Vladimir Vinogradov*

Date of Exam:	*September 18, 2003*
Time Period:	Start time: *7:00 pm* End Time: *8:00 pm*
Duration of Exam:	*1 hours*
Number of Exam Pages: (including this cover sheet)	*10*
Exam Type:	*Closed Book*
Additional Materials Allowed:	*Calculator*

QUESTION	VALUE	SCORE
1	20	
2	20	
3	15	
4	25	
5	20	
TOTAL	100	

1. (20 points) Differentiate

a) $f(x) = e^{3 \log_3 x}$

ANSWER: ____________________

b) $g(x) = \dfrac{\ln(x+1)^2}{2(x+1)^2}$

ANSWER: ______________________

2. (20 points) Integrate

a) $\displaystyle\int \frac{(x^2+1)e^{x(x^2+3)}}{e^{x(x^2+3)}+5}dx$

ANSWER: ____________________

b) $\int\limits_{1}^{e^2} \frac{\log_2(x^{\ln 2})}{x} dx$

ANSWER: ______________________

3. (15 points) Solve the differential equation

$$\frac{dy}{dx} = -k\,(x-a)\,y, \qquad y(a) = 1$$

where a is a constant.

ANSWER: ______________________

4. (25 points)

a) Solve the differential equation

$$y' + y = e^x.$$

Determine the constat of integration using the fact that the function is even $y(x) = y(-x)$.

ANSWER: ____________________

b) What if the function is odd $y(x) = -y(-x)$

ANSWER: ______________________

c) If y_a and y_b are the solutions of a) and b) respectively, show that

$$y_a' = y_b, \quad \text{and} \quad y_b' = y_a$$

5. (20 points) Solve the initial value problems

$$xy' + y = xe^{x^2}, \qquad y(1) = e/2$$

Useful formulae

Product rule:

$$\big(f(x)g(x)\big)' = f'(x)g(x) + f(x)g'(x)$$

Quotient rule:

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Chain rule:

$$\frac{d}{dx}f(g(x)) = \frac{df}{dg}\cdot\frac{dg}{dx}$$

$$\log_a x = \frac{\ln x}{\ln a}, \qquad a^x = e^{x\ln a},$$

$$\log_a x^n = n\log_a x, \qquad a^b a^c = a^{b+c},$$

$$(x^\alpha)' = \alpha x^{\alpha-1}, \qquad (a^x)' = a^x \ln a,$$

$$(\ln x)' = \frac{1}{x}, \qquad \int \frac{1}{x}dx = \ln x + C$$

$$(e^{f(x)})' = e^{f(x)}f'(x)$$

$$\int e^{f(x)}f'(x)dx = \int e^u du = e^{f(x)} + C$$

$$\int e^{kx}dx = \frac{e^{kx}}{k} + C$$

$$\int f^\alpha(x)f'(x)\,dx = \frac{f(x)^{\alpha+1}}{n+1} + C$$

$$y'(x) + P(x)y(x) = Q(x) \quad \Rightarrow \quad \left[e^{\int P(x)dx}y(x)\right]' = Q(x)e^{\int P(x)dx}$$