RESEARCH STATEMENT

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Contents

My research focuses on studying the geometry and combinatorics of objects that appear in representation-theoretic contexts. Much of my work is centered on extracting representation theoretic information about a semi-simple Lie group from the geometry of its nilpotent cone, and other Springer theoretic varieties. While such varieties were first introduced to study representations of the Weyl group, more recent work has established links with modular representations of Lie algebras, categorification, affine Hecke algebras and category O. In particular, I am interested in exotic t-structures on categories of coherent sheaves on Springer theoretic varieties, which were recently introduced by Bezrukavnikov and Mirkovic to study representation theory of Lie algebras in positive characteristic; and in the geometry of the exotic nilpotent cone, which was introduced by Kato as a substitute for the type C nilpotent cone with better properties. I have also worked on describing stability conditions for category O, and construction of crystal bases for quantum groups using Lusztig's quiver varieties (following Kashiwara-Saito). In this statement, I will summarize completed work, and outline related projects which I intend to work on in the future.

1. Modular representation theory, and categorification

This strand of my research focuses on constructing categorification results using blocks of categories of modular representations, and computing combinatorial character/dimension formulae for the irreducible modules in these blocks. The main ingredients are [BMR] localization theory (which relates these categories to categories of coherent sheaves), and geometric categorification results proven by Cautis, Kamnitzer and Licata. The resulting categorifications that we obtain are positive characteristic analogues of those constructed by Bernstein-Frenkel-Khovanov, Stroppel, Sussan, et al. using blocks of category $\mathcal{O}$.

The set-up in Bezrukavnikov, Mirkovic and Rumynin is as follows (see [9], [10]): Let $\mathfrak{g}$ be a semi-simple Lie algebra over a field $\mathbf{k} = \overline{\mathbf{k}}$ of characteristic p sufficiently large. Let $\lambda \in \mathfrak{h}_{\mathbf{k}}$ be integral, regular; and let $e \in \mathcal{N}(\mathbf{k})$ be a nilpotent. Let $\mathrm{Mod}_e^{fg,\lambda}(U_{\mathbf{k}})$ be the category of modules with generalized central character (λ, e). Theorem 5.3.1 from [10] (see also Section 1.6.2 from [9]), states that there is an equivalence, which sends the tautological t-structure on the LHS to the "exotic" t-structure on the RHS:

$$D^b(\mathrm{Coh}_{\mathcal{B}_{e,\mathbf{k}}}(\widetilde{\mathfrak{g}}_{\mathbf{k}})) \simeq D^b(\mathrm{Mod}_e^{fg,\lambda}(U_{\mathbf{k}}))$$

1.1. Two-block nilpotents.

Previous work: In [3] (joint with Rina Anno), we study exotic t-structures in type A, for nilpotents whose Jordan type has two blocks; our main result is an explicit description of the irreducible objects in the heart of these t-structure. Let $\mathfrak{g} = \mathfrak{sl}_{m+2n}$, let z be the standard nilpotent of type $(m+n, n)$. The Springer fiber is the variety

$$\mathcal{B}_z = \pi^{-1}(z) = \{(0 \subset V_1 \subset \cdots \subset V_{m+2n} = \mathbb{C}^{m+2n}) \mid zV_i \subseteq V_{i-1}, \dim V_i = i\}$$

We will be studying the exotic t-structures on the categories $\mathcal{D}_n = D^b(\mathrm{Coh}_{\mathcal{B}_z}(U_z))$ (the bounded derived category of coherent sheaves on the resolution of the Slodowy slice, U_z, supported on $\mathcal{B}_z$). Using techniques of Cautis and Kamnitzer from [12], in Theorem 4.26 of [3], we prove that:

Theorem 1.1. Let $\mathbf{ATan}_m$ be the category with objects $\{m+2k\}$ for $k \in \mathbb{Z}_{\geq 0}$, and the morphisms between $\{m+2p\}$ and $\{m+2q\}$ consist of all affine $(m+2p, m+2q)$ tangles (ie. a diagram with $m+2p$ points on an inner circle, $m+2q$ points on an outer circle, arcs connecting these endpoints, and a finite number of circles). Then there is a weak representation of the category $\mathbf{ATan}_m$ using the categories $\mathcal{D}_k$ (ie. for each affine $(m+2p, m+2q)$ tangle α there is a functor $\Psi(\alpha) : \mathcal{D}_p \to \mathcal{D}_q$, and an isomorphism $\Psi(\beta) \circ \Psi(\alpha) \simeq \Psi(\beta \circ \alpha)$ for each $(m+2q, m+2r)$-tangle β).

We show some compatibility results between this action, and the $\mathbb{B}_{aff}$-action which can used to define exotic t-structures. This allows to give the following description of the simple objects (see Proposition 5.10 in [3]); see also Theorem 5.15 of [3], where give explicit combinatorial descriptions of Ext's between the simples, and show that the resulting Ext algebra is an annular version of Khovanov's arc algebra.

Theorem 1.2. Let $\mathrm{Cross}(n)$ be the set of affine $(m, m+2n)$ tangles with n cups, no crossings, and inner points unlabelled. For each $\alpha \in \mathrm{Cross}(n)$, we have a functor $\Psi(\alpha) : \mathcal{D}_0 \to \mathcal{D}_n$; let $\Psi_\alpha = \Psi(\alpha)(\underline{\mathbb{C}})$ (noting that $\mathcal{D}_0 \simeq D^b(\mathrm{Vect})$). Then $\{\Psi_\alpha \mid \alpha \in \mathrm{Cross}(n)\}$ constitute the irreducible objects in $\mathcal{D}_n^0$.

1.2. Ongoing, and future work.

Modular representation theory, and Koszulity of annular arc algebras. In an ongoing project (see [23]), we re-interpret the above result on the RHS of the [BMR] equivalence. These simple exotic sheaves correspond to irreducible representations in $\mathrm{Mod}_e^{fg,\lambda}(U_{\mathbf{k}})$. In Proposition 2.8 of [23], we give combinatorial dimension formulae for these irreducible modules, by working on the level of the Grothendieck group, and using the fact (see Section 6.2 of [10]) that the dimension of an irreducible module M in the Euler characteristic of $\mathrm{Fr}^*\mathcal{F}_{\mathcal{M}} \otimes \mathcal{O}(\lambda)$ (here $\mathcal{F}_M$ is the corresponding exotic sheaf). We will also compute character formulae using similar techniques, and to re-interpret Theorem 1.1 above using the categories $\mathrm{Mod}_e^{fg,\lambda}(U_{\mathbf{k}})$; this is a positive characteristic analogue of the tangle categorification using blocks of parabolic category $\mathcal{O}$ due to Bernstein, Frenkel and Khovanov ([4]), and Stroppel ([30]).

In another ongoing project, we will give a combinatorial re-formulation of Theorem 1.1 using the annular arc algebras mentioned above (following Khovanov's construction in [18]); this will give an extension of Khovanov homology to links in an annulus (and more generally, affine tangles). We will adopt Khovanov's TQFT approach, but using "nested co-bordisms" introduced by Stroppel and Webster in [31]. We will also show that the Koszul dual algebra is isomorphic to an annular version of the KLR-type algebra defined by Webster in [32]; this will essentially follow once we construct a categorification of the affine tangle calculus using these algebras, and compare it to Theorem 1.1. It will then follow that $\mathrm{Mod}_e^{fg,\lambda}(U_{\mathbf{k}})$ is equivalent to the category of modules over this annular KLR-type algebra (which is a positive characteristic analogue of Webster's diagrammatic description of two-block singular category $\mathcal{O}$ in [32]).

We also expect that it will be possible to extend some of these results for two-block nilpotents to other nilpotents in type A. The main technical tool we used here is Cautis-Kamnitzer's categorification of the intertwiners between $\mathfrak{sl}_2$ representations in [12]; this is generalized to $\mathfrak{sl}_k$ in [13]. While we no longer expect that the same method will produce all irreducible exotic sheaves for the nilpotent in question, we do expect that they will give some of them. It will then possible to extract some representation theoretic information about the corresponding irreducible representations.

Categorification via singular blocks of modular representations

In ongoing work with Gufang Zhao (see page 2 of [22] for the main result), we give a characteristic p analogue of Bernstein-Frenkel-Khovanov's categorification of the $\mathfrak{sl}_2$-action on $(\mathbb{C}^2)^{\otimes n}$ using singular category $\mathcal{O}$. Let $\mathfrak{g} = \mathfrak{sl}_n$; the $(-n+2l)$-weight space in $(\mathbb{C}^2)^{\otimes n}$ is categorified by $\mathrm{Mod}^{fg}_{\mu_l,0}(\mathbf{U}\mathfrak{g}_{\mathbf{k}})$ where $\mu_l = -\rho + e_1 + \cdots + e_l$ is specifies the singular Harish-Chandra character, and 0 specifies the action of the p-center. As in [BFK] (see [4]), the maps E and F between these weight spaces are categorified by the translation functors between these blocks; we first check the $\mathfrak{sl}_2$ relations on the level of Grothendieck groups (by verifying them on the spanning set given by Weyl modules). To finish the proof, we show this constitutes a $\mathfrak{sl}_2$-categorification in the sense of Chuang and Rouquier, by checking their criterion.

We plan to show this corresponds to a geometric categorification constructed by Cautis, Kamnitzer, and Licata, action after it's transported to the other side under a certain [BMR]-type localization result proven in [24] (which shows that $\mathrm{Mod}^{fg}_{\mu_l,0}(\mathbf{U}\mathfrak{g}_{\mathbf{k}})$ is derived equivalent to the category of coherent sheaves on the co-tangent bundle to a partial flag variety). The categories appearing in Riche's equivalence have Koszul gradings, and this will give a graded version of the above. We also plan to generalize this to give a categorification of the $\mathfrak{sl}_k$-action on $(\mathbb{C}^k)^{\otimes n}$, which should be fairly straightforward. In the [BFK] framework, more generally one categorify tensor products of wedge powers (as done in Sussan's thesis), using parabolic blocks of singular category $\mathcal{O}$. We expect that it is possible to construct a positive characteristic analogue of this; however, the corresponding geometric statement seems to be much more delicate.

2. Combinatorial bijections arising from Springer theory

Many aspects of the representation theory of a semisimple Lie group can be seen by studying the geometry of its nilpotent cone. However, the Springer correspondence in types B, C and D is more complicated than that in type A in many ways. Kato has introduced the "exotic nilpotent cone", as a substitute for the nilpotent cone in type C which mimicks its behaviour and in many cases has better properties.

Let $G = Sp_{2n}(\mathbb{C})$ and $\mathfrak{g} = \mathfrak{sp}_{2n}(\mathbb{C})$ be the symplectic group and its Lie algebra (here $\mathbb{C}^{2n}$ is equipped with a symplectic form). Kato's exotic nilpotent is defined to be $\mathfrak{N} = \mathbb{C}^{2n} \times \mathfrak{N}_0$ where:

$$\mathfrak{N}_0 = \{x \in \mathrm{End}(\mathbb{C}^{2n}) \mid x \text{ nilpotent}, \langle xv, v\rangle = 0 \ \forall\ v \in \mathbb{V}\}$$

In [17], Kato shows that the G-orbits on $\mathfrak{N}$ are in bijection with $\mathcal{Q}_n$ (the set of bi-partitions of n), and constructs an analogue of the type C Springer correspondence using the exotic nilpotent cone which is cleaner than the type C Springer correspondence. In [16], Kato has studied multi-parameter affine Hecke algebras using the equivariant K-theory of the exotic Steinberg variety (following techniques used by Kazhdan, Lusztig and Ginzburg).

2.1. **Previous work.** Now let G be any semisimple algebraic group; denote by $\mathfrak{g}$ its Lie algebra and $\mathcal{N} \subset \mathfrak{g}$ its nilpotent cone. Let Λ^+ denote the set of dominant weights for G, and $\mathbf{O}$ denote the set of pairs $(\mathcal{O}, V)$, where $\mathcal{O}$ is a G-orbit on $\mathcal{N}$, and V is a finite-dimensional irreducible representation of the isotropy group G^x of the orbit $\mathcal{O}$, where $x \in \mathcal{O}$. Motivated in part by the theory of two-sided cells in affine Weyl groups, Lusztig and Vogan conjectured (independently) that there is a canonical bijection between Λ^+ and $\mathbf{O}$. Using geometric methods, in [8], Bezrukavnikov proves this conjecture by studying the irreducible objects in a certain t-structure on $D^b(Coh^G(\mathcal{N}))$. It is also shown in [7] that on the level of Grothendieck groups, the natural map $D^b(Coh^{G\times\mathbb{C}^*}(St)) \to D^b(Coh^{G\times\mathbb{C}^*}(\mathcal{N}))$ sends a canonical basis element in $\mathbf{H}_{aff}$ either to 0, or to the class of an irreducible object.

Now specialize to $G = Sp_{2n}(\mathbb{C})$; in [19], we follow the method used in [8] to establish an exotic analogue of this bijection. Let $\mathbb{O}$ be the set of pairs $(\mathscr{O}, L)$, where $\mathscr{O}$ is a G-orbit on $\mathfrak{N}$, and L is a finite-dimensional irreducible representation of the isotropy group G^x of the orbit $\mathscr{O}$ (here $x \in \mathscr{O}$). In Theorem 4.8 of [19] we construct a bijection between Λ^+ and $\mathbb{O}$.

The quasi-exceptional t-structure on $D^b(Coh^G(\mathfrak{N}))$: Let $\mathcal{C} = D^b(\mathrm{Coh}^G(\mathfrak{N}))$. In [16], [17], Kato constructs a semi-small resolution of singularities $\pi : \widetilde{\mathfrak{N}} \to \mathfrak{N}$; we also have the vector bundle projection $p : \widetilde{\mathfrak{N}} \to G/B$. Let $\mathcal{O}_{\widetilde{\mathfrak{N}}}(\lambda) = p^*\mathcal{O}_{G/B}(\lambda)$, and denote $\nabla_\lambda = R\pi_*\mathcal{O}_{\widetilde{\mathfrak{N}}}(\lambda)[d]$ where $d = \frac{\dim(\mathfrak{N})}{2}$. In Section 2 of [19], we describe the cohomology of the sheaves $\mathcal{O}_{\widetilde{\mathfrak{N}}}(\lambda)$. In Theorem 3.16 of [19], we prove that:

Theorem 2.1. There is a unique t-structure on $\mathcal{C} = D^b(\mathrm{Coh}^G(\mathfrak{N}))$, such that $\nabla_\lambda \in \mathcal{C}^{\geq 0}$ and $\nabla_{w_0\cdot\lambda} \in \mathcal{C}^{\leq 0}$ for $\lambda \in \Lambda^+$.

To prove this, we appeal to the well-established theory of a quasi-exceptional sets in a triangulated category $\mathcal{C}$; (very similar to that of a highest weight category). It follows general theory that the simple objects in the heart are also naturally indexed by Λ^+. Meanwhile, applying the theory of perverse coherent t-structures (see [6]), one obtains another t-structure on $D^b(\mathrm{Coh}^G(\mathfrak{N}))$. From the general theory, the irreducible objects in the heart $\mathcal{P}$ of this t-structure are parametrized by $\mathbb{O}$ (or, more generally, with a irreducible vector bundle over a G-orbit). In Proposition 4.6 of [19], we prove that:

Theorem 2.2. The perverse coherent t-structure on $D^b(\mathrm{Coh}^G(\mathfrak{N}))$ coincides with the above quasi-exceptional t-structure. Comparing the parametrization of irreducible objects, the desired bijection between Λ^+ and $\mathbb{O}$ follows.

2.2. **Future work.**

In [1], Achar gives an explicit description of the bijection in type A using some complicated combinatorial algorithms. In future work, I plan to work towards giving a similar description of the bijection constructed above. After we restrict to pairs on the RHS where the nilpotent orbit lies in $\mathfrak{N}_0 \subset \mathfrak{N}$, the bijection should become more tractable, since in this case, the resolutions of these nilpotent orbits are known (and Achar uses these resolutions in an essential way). This is also closely related to a generalization of the Lusztig-Vogan conjecture to real groups, for the pair $(\mathfrak{g}, \mathfrak{k})$ with $\mathfrak{g} = \mathfrak{sl}_{2n}, \mathfrak{k} = \mathfrak{sp}_{2n}$.

In an separate project, joint with Daniele Rosso and Neil Saunders, I plan to give a geometric construction of the type C Robinson-Schensted correspondence using the exotic nilpotent cone. Recall that in type A, motivated by the representation theory of S_n, the Robinson-Schensted correspondence gives an explicit combinatorial bijection between elements of S_n and and pairs of standard young tableau of fixed shape. In [29], Steinberg gives a geometric realization of this bijection by parametrizes the irreducible components of the Steinberg variety $\mathbf{St} = \widetilde{\mathcal{N}} \times_{\mathcal{N}} \widetilde{\mathcal{N}}$ in two different ways, with one giving the LHS of the bijection, and the other giving the RHS of the bijection.

While this bijection can be generalized to arbitrary semi-simple Lie groups, in the other classical types the bijection differs from the version of the Robinson-Schensted correspondence one expects from studying representations of the Weyl group. However, in type C, by instead studying the geometry of the exotic Steinberg variety $\underline{\mathbf{St}} = \widetilde{\mathfrak{N}} \times_{\mathfrak{N}} \widetilde{\mathfrak{N}}$, we expect to recover the type C Robinson-Schensted correspondence.

3. Quantum groups, category O

Stability conditions for category O. Inspired by Bridgeland's theory of stability conditions, in [2], Anno, Bezrukavnikov and Mirkovic define the notion of a "real variation of stability conditions" on a triangulated category $\mathcal{C}$ (consisting of a collection of t-structures on it, indexed by connected components in a given hyperplane arrangement in a vector space V, and a "central charge" map $Z : V \to (K^0(\mathcal{C}) \otimes \mathbb{R})^*$, satisfying some compatibilities). They then give an example of this construction, with $\mathcal{C} = D^b(\mathrm{Coh}_{\mathcal{B}_e}(\widetilde{S}))$ (where $\widetilde{S}$ is the pre-image to the Slodowy slice at e under the Springer map), with the hyperplane arrangement is the set of affine co-root hyperplanes in $V = \mathfrak{h}^*_{\mathbb{R}}$; the central charge map involves taking the Euler characteristic of tensor products of sheaves with line bundles, the t-structures on are the exotic t-structures from [9].

In [21], I construct another example using certain sub-quotients of the principal block $\mathcal{O}_0$ of BGG category $\mathcal{O}$ for a semi-simple Lie algebra $\mathfrak{g}$. For $d \in \mathbb{Z}_+$, let $\mathcal{O}_0^{<\mathbf{d}}$ (resp. $\mathcal{O}_0^{\leq \mathbf{d}}$) be the Serre subcategories consisting of objects with Gelfand-Kirillov dimension less than (resp. less than or equal to) d; we will be working with the derived quotients $\mathcal{C} = D^b(\mathcal{O}_0^{\mathbf{d}}) = D^b(\mathcal{O}_0^{\leq \mathbf{d}}/\mathcal{O}_0^{<\mathbf{d}})$, with the hyperplane arrangement being $V = \mathfrak{h}^*_{\mathbb{R}}$ and Σ being the set of linear co-root hyperplanes. Given $\lambda \in \Lambda^+, M \in \mathcal{O}_0^{\mathbf{d}}$, the central charge map Z is defined by letting $Z(\lambda)[M]$ to be leading coefficient in a certain filtration polynomial for the module $T_{0\to\lambda}M$, and the t-structures come from the action of the braid group $\mathbb{B}_W$ on $D^b(\mathcal{O}_0^{\mathbf{d}})$.

Quiver varieties and the $B(\infty)$ crystal. Let $\mathfrak{g}$ be a symmetrizable Kac-Moody algebra, and $U_q(\mathfrak{g})$ the corresponding quantum group. The $B(\infty)$ (and $B(\lambda)$) crystals are combinatorial gadgets which encode much of the representation theory of $U_q(\mathfrak{g})$ (for instance, tensor product multiplicities). In [15], Kashiwara and Saito geometrically realize the $B(\infty)$ crystal as irreducible components of Lusztig's quiver varieties.

We briefly recall their construction. Let Γ be a directed graph with vertex set I, and k edges between vertices i and j, if $\langle \check{\alpha}_i, \alpha_j \rangle = -k$. The pre-projective algebra Λ is the quotient of the path algebra $\mathbb{C}\Gamma$ by the ideal generated by certain pre-projective relations for each vertex. Let $\Lambda(\nu)$ consist of representations of Λ with weight vector $\nu \in Q^+ = \sum_{i \in I} \mathbb{Z}_{\geq 0} \alpha_i$. One of the main results in [15] is that the set

$$\bigsqcup_{\nu \in Q^+} \mathrm{Irr}\ \Lambda(\nu)$$

can be equipped with the structure of a crystal, and is isomorphic to $B(\infty)$.

3.1. Previous work.

In [20] (joint with Peter Tingley), we extend this result to arbitrarily symmetrizable Lie algebras $\mathfrak{g}$. We consider a directed graph Γ with the additional datum of a finite field extension $\mathbb{F}_i$ of $\mathbb{Q}$ for each vertex $i \in I$; and a $(\mathbb{F}_i, \mathbb{F}_j)$ bi-module for each edge τ between node i and node j. Given a modulated graph, we can extract the Cartan datum and consider the corresponding symmetrizable Kac-Moody algebra $\mathfrak{g}$. We can also define the path algebra $\mathbb{C}\Gamma$ by taking tensor products of bi-modules, and define the pre-projective algebra Λ as the quotient of $\mathbb{C}\Gamma$ by certain canonical elements. Define $\Lambda(\nu)$

to be representations of Λ with weight vector ν. Analogously to the above, in Theorem 4.7 of [20] we show that

$$\bigsqcup_{\nu \in Q^+} \text{Irr } \Lambda(\nu)$$

can be equipped with the structure of a crystal, and is isomorphic to $B(\infty)$.

3.2. **Future work.** In future work, inspired by Savage's paper [26] (in type A), we plan to make a connection between the geometric realization of crystal bases constructed above, and the known combinatorial realizations in types B and C. In [25], Saito gives a geometric construction of the crystal $B(\lambda)$ for the highest weight irreducibles of $U_q(\mathfrak{g})$ (where $\mathfrak{g}$ is a symmetric Kac-Moody Lie algebra), by considering irreducible components of Nakajima's quiver varieties. It would be interesting to extend this result to arbitrary symmetrizable Kac-Moody algebras, by using the techniques developed in [20]. This would be a first step towards giving a construction of Nakajima's quiver varieties in non-symmetric type.

One caveat of the above construction is that it does not work over algebraically closed fields, since we need to use field extensions of $\mathbb{F}$; when we base change to its algebraic closure, the resulting varieties no longer have the correct properties. It would be interesting to use this idea to construct quiver varieties over $\mathbb{C}$ in non-symmetric type, by making some modifications to these.

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