SMOOTH MANIFOLDS FALL 2022 - MIDTERM

Problem 1. Let M be a C^∞ manifold and $i_1 : N_1 \to M$ be an embedding. Show that if $i_2 : N_2 \to M$ is another embedding such that $i_1(N_1) = i_2(N_2)$, then N_1 and N_2 are diffeomorphic.

Problem 2. Consider the function $f(x, y, z) = z^2 - x^2 - 2y^2$. Find the regular values of f. Find the values of $r > 0$ such that the cylinder $x^2 + z^2 = r^2$ intersects $f^{-1}(1)$ transversally. Justify your answers.

Problem 3. Let φ_t, ψ_s, and η_u be fixed-point free flows on a C^∞ manifold M whose generating vectors fields are linearly independent at every point. Assume that η_u commutes with both φ_t and ψ_s, and that there exists a C^∞ function $\sigma : \mathbb{R}^2 \times M \to \mathbb{R}$ such that $\sigma(0, t, p) = 0$ and $\sigma(s, 0, p) = 0$ for all $s, t \in \mathbb{R}$ and $p \in M$ and

$$\varphi_t \psi_s(p) = \eta_{\sigma(s,t,p)} \psi_s \varphi_t(p).$$

Find a 3-dimensional foliation in M containing the orbits of φ_t, ψ_s and η_u. Be sure to prove that it is a foliation!