MATH3210(001) - Quiz 1 *Solution*
February 9, 2024

Proofs and justifications should be written in **complete sentences** with **correct logical flow**.

Problem 1. Using the definition of a limit (and not the Limit Arithmetic Theorem), show that if $a_n = \dfrac{\sqrt{n}}{\sqrt{n}+1}$, then $a_n \to 1$.

Proof. Let $\varepsilon > 0$. Then choose any natural number N such that $N > (1/\varepsilon - 1)^2$ which is possible by the Archimedean property. With this choice, if $n \geq N$,

$$\begin{aligned}
n &> (1/\varepsilon - 1)^2 \\
\sqrt{n} &> 1/\varepsilon - 1 \qquad \text{[both positive]} \\
\sqrt{n} + 1 &> 1/\varepsilon \\
\frac{1}{\sqrt{n}+1} &< \varepsilon \qquad \text{[both posiitve]} \\
\left|\frac{1+\sqrt{n}-\sqrt{n}}{\sqrt{n}+1}\right| &< \varepsilon \\
\left|\frac{\sqrt{n}}{\sqrt{n}+1} - 1\right| &< \varepsilon
\end{aligned}$$

Hence by definition of sequence convergence, $a_n \to 1$. □