MATH3210 - SPRING 2024 - SECTION 004

HOMEWORK 9

Problem 1 (30 points)**.** Let $a < d_1 < d_2 < \cdots < d_k < b$ be a finite list, $g_i : [d_i, d_{i+1}] \to \mathbb{R}$ be a continuous function for every $i = 1, \ldots, k$, and $f : [a, b] \to \mathbb{R}$ be the function defined by

$$f(x) = g_i(x) \qquad \text{when } x \in [d_i, d_{i+1})$$

and $f(b) = g_k(b)$. Show that f is integrable.

Problem 2 (30 points)**.** Let u and v be continuously differentiable functions on $[a, b]$, and V be an antiderivative of v. Show that

$$\int_a^b uv\,dx = u(b)V(b) - u(a)V(a) + \int_a^b Vu'\,dx.$$

[*Hint*: Apply the fundamental theorems to the fuction $H(y) = \displaystyle\int_a^y uv\,dx$]

Problem 3 (40 points)**.** Let $f_n : (a, b) \to \mathbb{R}$ be a sequence of functions on an open interval (a, b). For each, prove or find a counterexample:

(a) If each f_n is bounded, and $f_n \to f$ pointwise, then f is bounded.
(b) If each f_n is bounded, and $f_n \to f$ uniformly, then f is bounded.