MATH3210 - SPRING 2024 - SECTION 004

HOMEWORK 7

Problem 1 (20 points)**.** Prove that if f is defined on (a, b) is differentiable at c, $f(c) \neq 0$, and $g(x) := 1/f(x)$, then $g'(c) = -\dfrac{f'(c)}{f(c)^2}$.

Problem 2 (80 points)**.** For each, either calculate $f'(0)$ with justification, or prove that f is not differentiable at 0. You may assume continuity and the usual properties and formulas for the function sin. [*Hints*: Try to sketch a graph if you can to get an idea. The points $x_n = 1/(2\pi n)$ are especailly useful in the graph and proofs for (c) and (d). The squeeze theorem is useful!]

(a) $f(x) = \begin{cases} 0, & x < 0 \\ x^2, & x \geq 0 \end{cases}$

(b) $g(x) = \begin{cases} 0, & x < 0 \\ x, & x \geq 0 \end{cases}$

(c) $h(x) = \begin{cases} 0, & x = 0 \\ x \sin(1/x), & \text{otherwise} \end{cases}$

(d) $k(x) = \begin{cases} 0, & x = 0 \\ x^2 \sin(1/x), & \text{otherwise} \end{cases}$

Extra practice. Show that each of the functions in the last problem are all continuous. One of them has a very interesting property: the function is differentiable at 0, but the derivative as its own function is not continuous at 0. Find which one it is, and prove your answer.