MATH3210 - SPRING 2024 - SECTION 004

HOMEWORK 6

Let f be real-valued function whose domain is a subset of the real numbers. We say that f is L-Lipschitz if for every pair of points x, y in the domain of f, $|f(x) - f(y)| \leq L\,|x - y|$.

Problem 1 (80 points)**.** Prove or find a counterexample for each:

(a) If f is uniformly continuous, then f is L-Lipschitz for some $L > 0$
(b) If f is L-Lipschitz for some $L > 0$, then f is uniformly continuous
(c) If f and g are L-Lipschitz, then there exists an L' such that $f + g$ is L'-Lipschitz
(d) If f is L-Lipschitz, then there exists some L' such that $g(x) := f(x)^2$ is L'-Lipschitz

Problem 2 (20 points)**.** Show that if $f : [a, b] \to [c, d]$ is continuous and has an inverse, then either f is increasing or f is decreasing.