MATH3210 - SPRING 2024 - SECTION 004

HOMEWORK 5

Problem 1 (40 points)**.** Let $D \subset \mathbb{R}$ be a domain of functions f and g, and $x \in D$. Show *directly* that if f and g are continuous at x, then $f + g$ is continuous at x. *Do not use any theorems, use the ε-δ definition of continuity.*

Problem 2 (20 points)**.** Prove or find a counterexample: if $I = (a, b) \subset \mathbb{R}$ is an open interval and $f : I \to \mathbb{R}$ is a continuous function, then the image of f is an open interval.

Problem 3 (40 points)**.** Let f and g be functions defined on the open interval $(-1, 1)$, and assume that f and g are continuous at 0. Show that the function h defined by

$$h(x) := \max \{f(x), g(x)\}$$

is continuous at 0.