# MATH3210 - SPRING 2024 - SECTION 001

HOMEWORK 2

**Problem 1.** Consider the expression $x = .99999\dots$, which we think of as a decimal expansion. That is, if $x_n = \sum_{k=1}^{n} 9 \cdot 10^{-k}$, we think of it as

$$x = \lim_{n\to\infty} .9999\dots 9 = \lim_{n\to\infty} x_n.$$

Show that $x = 1$. [*Hint*: First show that $x_n + 10^{-n} = 1$ by induction]

**Problem 2.** Consider the following property for a set $A \subset \mathbb{R}$.

(N) If $x \in A$, then there exists some $\varepsilon > 0$ such that if $|x - y| < \varepsilon$, then $y \in A$.

Show that if $A$ is a bounded subset of $\mathbb{R}$ with property (N), then $\sup A \notin A$.

**Problem 3.** Show that if $(a_n)$ is increasing, then for every $N \in \mathbb{N}$, if $n \geq N$, $a_n \geq a_N$. [*Hint:* Use induction]

**Problem 4.** Assume $(a_n)$ is increasing. Show that if $(a_n)$ is not bounded above if and only if $(a_n)$ diverges to $\infty$.

**Problem* 5** (Digit Expansions, Extra Credit)**.** Let $\Sigma_{10}$ denote the set of infinite sequences in the digits $\{0, 1, \dots, 9\}$. Denote an element of $\Sigma_{10}$ by $\sigma = (\sigma_1, \sigma_2, \dots)$, where each $\sigma_i \in \{0, 1, \dots, 9\}$. If $\sigma \in \Sigma$, let $(a(\sigma)_n)$ denote the sequence

$$a(\sigma)_n = \sum_{k=1}^{n} \sigma_i \cdot 10^{-k}.$$

Show that $(a(\sigma)_n)$ converges. [*Hint*: Use Problem 1 and the Monotone convergence theorem]