Midterm
Math 1030 - 1

1. (5 pts) Express the following number in scientific notation. Remember that scientific notation of a number consists in writing it as a number **between 1 and 10** times a power of ten.

   Seventy one million four hundred thousand = $\boxed{7.14 \times 10^7}$

2. (10 pts) How many square feet are in a square yard? Explain using a picture or by using algebra.

   $3ft = 1yd$ so by squaring both sides of the equation we get $1yd^2 = (3ft)^2 = \boxed{9ft^2}$

3. (10 pts) Compute the number of seconds in a week by making a chain of conversions.

   $$\frac{7 \text{ days}}{1 \text{ week}} \times \frac{24h}{1\text{day}} \times \frac{60min}{1h} \times \frac{60s}{1min} = 604,800s/week$$

4. (10 pts) You recently went to a basketball game. You paid \$2 for a soda at the concession stand. There was 6.35% sales tax included in the price. What was the price of your drink before tax?

   Let x denote the price before tax. The price after tax is $x + x \times 6.35/100 = x \times 1.0635$. *Since we know that the price after tax is* \$2 *we get the equation* $1.0635 \times x = 2$ *so* $x = 2/1.0635 = 1.88058$ *which we may roud at* $x = \$1.88$.

   Note: if you remove 6.35% from \$2, you get a different amount of \$1.873. This is because of the *shifting in the value*. The two percent are not computed on the price with tax but on the pretax price. If up to rounding, you found the same amount, that's just luck (that's actually because the tax rate is low). There would be more difference for a tax rate of 20% (like in France, where it is 20.6%).

5. (10 pts) You are considering two different savings accounts. Account 1 offers an APR of 5% with quartly compounding, and account 2 offers an APR of 4.9% with daily compounding. Decide which account you should choose by making a dummy deposit, and let it grow for one year. (A computation is required to get credit).

   You had to use the compound interests formula $A = P(1 + \frac{APR}{n})^{nY}$. *For an amount of \$100, you get after one year*

Account 1	*Account 2*
$100 \times (1 + \frac{.05}{4})^4 = \$105.09$	$100 \times (1 + \frac{.049}{365})^{365} = \$105.02$

   Therefore, the first account is a little better (but very little).

   Note: If you used the formula for savings plans, which gives you the amount you get when you put a regular deposit (n times a year) on the account, it wouldn't be correct to use the same PMT (say 100\$) for the two accounts: because this way, you would put \$400 for on first one, and \$36500 on the other one. Of course, you would get more on account number 2 this way but this is not because the account is better.

6. (10 pts) You put \$5000 in an account at 9% APR with monthly compounding. How long will it take for you to have \$8000 in your account? Answer in years and months.

   Here, you had to use the formula for compound interests. (If you used the formula for saving plans, setting PMT=5000. it meant that you put \$5000 each mont on the account. In this case it is not hard to guess that you need 2 months to get \$8,000. But that does not answer the question) The equation is $8000 = 5000 \times (1 + \frac{.09}{12})^{12Y}$ *which is* $8000 = 5000 \times 1.0075^{12Y}$. *Dividing by 5000, we get* $1.6 = 1.0075^{12Y}$. *Taking logs and bringing the exponent down, we obtain* $\log 1.6 = 12Y \log(1.0075)$ *So* $Y = \frac{\log(1.6)}{12 \log(1.0075)} = 5.24 years =$ $\boxed{\textit{5 years and 3 months}}$.

7. (10 pts) You can afford \$250 per month for your car payment. The loan you will take has an APR of 8% for 5 years. What is the most expensive car that you can afford (how much can you borrow)?

   The formula for loans give you the equation

   $$250 = P \times \frac{.08/12}{1 - (1 + .08/12)^{-60}} = P \times 0.02028$$

   so $P = 250/0.02028 =$ $\boxed{\textit{\$12,329}}$

8. (20 pts) The following data was issued by the Bureau of Economical Analysis (the table shows the Utah and US population for the given years, expressed in thousands of people).

Population	1993	1997
Utah	1,875	2,059
US	257,753	267,636

(a) Considering only the year 1997, express the Utah population as a percentage of the total US population.
In 1997, the population of Utah was $\frac{2059}{267.636} = 0.769\%$ *of the population of the United States.*

(b) Analyse the change in the US population from 1993 to 1997 by giving two sentences expressing its absolute change and its percent change. Numbers with no sentence won't get full credit.
Absolute change $= 267,636 - 257,753 =$ *9,883 million people. Percent change* $= \frac{9883}{257,753} = 3.8\%$.
The population of the United States increased by 9,883 million people between 1993 and 1997. In 1997, the population of the united states is 3.8% more than it was in 1993.

9. (15 pts) Using Moon-Telecom, you can phone to Jupiter at a the incredible rate of 35 cents a minute plus a one dollar connection fee (per call).

a. Identify the dependant and independant variables involved in the function linking the length of the call and its cost.
The independant variable is the length of the call, and the dependant variable is its cost.

b. Write a linear equation to represent this function.
Cost=\$1 + $0.35\$/min\times$ *length-of-the-call*

c. How long can you phone for 3.5 dollars ? *You solve for length of the call in the equation above:*

$$3.5 = 1 + .35 \times length$$

so $length = \frac{\$2.5}{.35\$/min} = 7.14 min = 7 min\, 8 s$.

10. (Extra credit) The Center of Demographical Studies of the Milky Way provides the following information about the population of Saturn:

Year (earth local time)	Population of Saturn (in million)
0 a.d*	50
1500 a.d.	100
2000 a.d.	500

Explain using the given data why the population of Saturn is not a linear function of time. Convincing argument required.

Well one good reason is that if you plot the three points on a graph, you see that they are not aligned, so it cannot be a linear function. Another good reason is that the rate of change is no the same if you compute it between 0 and 1500 or between 1500 and 2000.

*a.d. = Anno Domini = since Christ was born