Correction of Quiz 5

1. (3 pts) The number of inhabitants of Electre-city was 120,000 and increased to 160,000 in 10 years. Assume that the population grows linearly.

a. What is the rate of change of the population in Electre-city?

$$\textit{Rate of change} = \frac{\textit{Change of dependant}}{\textit{change of independant}} = \frac{160,000 - 120,000}{10} = \boxed{\textit{4,000 people/year}}$$

b. Assuming linear growth, when would the population reach 300,000 inhabitants?

After t years, the population will be $120,000 + 4,000 \times t$ so we just have to solve $120,000 + 4,000 \times t = 300,000$. We get $4,000 \times t = 300,000 - 120,000 = 180,000$ so

$$t = \frac{180,000}{4,000} = \boxed{\textit{45 years}}$$

2. (3 points) Your initial salary is \$1500 and if you keep the same job, you'll get a raise of 2% per year. Make a model describing how your salary will grow according to time. Make sure to label the variables you use. How much will you earn in 2010?

Call S your salary after t years. Since S is exponentially growing at a rate of 2% per year, S satisfies the equation $S = S_0 \times (1 + r)^t$ where $r = 2\% = .02$ and S_0 is the initial salary of \$1500. Thus we get the model

$$\boxed{S = 1500 \times (1.02)^t}$$

The year 2010 is 11 years from now so to get the salary in 2010, we plug $t = 11$ in the previous formula and we get $\boxed{1500 \times (1.02)^{11} = \$1865.06}$

3. (2 pts) The population of Perplex-city had an exponential growth for 55 years at a rate of growth of 2% per year. Today, it is a big city of 800,000 inhabitants. How much was the population 55 years ago?

Since the population had an exponential growth at a rate of growth of 2% per year, the population P satisfies the equation $P = P_0 \times (1.02)^5 5$ where P is the population today and P_0 is the initial population. Thus we solve for P_0 and we get

$$\boxed{P_0 = \frac{800,000}{(1.02)^{55}} = 269,000 \textit{ people}}$$

4. (2 points) A cloth made of wool contains only 34% of the carbon 14 it contained when the sheep was alive. Knowing that the half life of carbon 14 is 5,730 years, how old is the cloth (assuming that it has been knitted not too long after the death of the sheep)?

In t years, the quantity of Carbon 14 is $(\frac{1}{2})^{t/T_{half}}$ of what was originally, which is 34%. So we solve for t in the equation $(\frac{1}{2})^{t/5,730} = .34$. By taking logs and bringing the exponent down, we get $\frac{t}{5,730} \times \log(\frac{1}{2}) = \log(.34)$ so

$$\boxed{t = 5,730 \times \frac{\log(.34)}{log(.5)} = 8918 \textit{ years}}$$

5. (Extra credit) The golden bar you have in your safe happens to be radioactive. It was 1kg when you bought it, and 10 years later, there are only 900g left. What is the half life of your gold bar ?

The mass of radioactive gold remaining after 10 years is $1\times(\frac{1}{2})^{10/T_{half}}kg$ *which has to be 900g=.9kg. so we solve for* T_{half} *in the equation* $(\frac{1}{2})^{10/T_{half}} = .9$ *so by taking logs and bringing the exponent down, we get* $10/T_{half} \times \log(.5) = \log(.9)$ *then by multiplying by* T_{half} *and then dividing by* $\log(.9)$ *we get* $\boxed{t = 10 \times \frac{log(.5)}{\log(.9)} = 65.7 \text{ years.}}$