1 Example
For all types of problems, you need to give a clear step-by-step derivation that identify the names of the relevant theorems. In other words, you need to justify each step of moving towards the solution by theorems, lemmas, or previous knowledge.

Example 1. Prove the Maclaurin series of \( f(x) = \sin(x) \) converges \( \forall x \in \mathbb{R} \).

Proof. \( \forall x \in \mathbb{R}, \forall n \in \mathbb{N}, f^{(n)}(x) \leq 1 \). Identify \( M = 1 \) and \( a = 0 \) in Taylor’s inequality, we have \( |R_n(x)| \leq \frac{|x|^{n+1}}{(n+1)!} \).

By the ratio test, the series \( \sum_{n=0}^{\infty} \frac{x^n}{n!} \) converges. Hence \( \lim_{n \to \infty} \frac{|x|^{n+1}}{(n+1)!} = 0 \) by Theorem 6 on page 570 of the textbook. By the comparison test, \( \lim_{n \to \infty} |R_n(x)| = 0 \). Hence the Maclaurin series of \( f(x) = \sin(x) \) converges by Theorem 6 on the summary handout of §8.5 - §8.7.

Example 2. Express the indefinite integral \( \int \frac{1}{1-t^8} \, dt \) as a power series and then approximate \( \int_{1/2}^{1} \frac{1}{1-t^8} \, dt \) to six decimal places.

Solution. By the power series of \( \frac{1}{1-t} \),
\[
\frac{t}{1-t^8} = t \sum_{n=0}^{\infty} (t^8)^n = \sum_{n=0}^{\infty} t^{8n+1}.
\]
Then, by the theorem of term-by-term integration,
\[
\int \frac{t}{1-t^8} \, dt = C + \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2}.
\]
Setting \( t = 0 \) in the above equation yields \( C = 0 \).
\[
\int_{1/2}^{1} \frac{t}{1-t^8} \, dt = \left( \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2} \right)_{1/2}^{1} = \left( \frac{1}{2} + \frac{1}{10} + \frac{1}{18} + \ldots \right)_{1/2}^{1}
\]
\[
\approx \frac{1}{8} + \frac{1}{10240}. \tag{1}
\]
By the theorem on remainder estimate for integral test, (1) satisfies the accuracy requirement since
\[
R_1 = \frac{1}{18} \times 2^{18} \approx 2 \times 10^{-7} < 10^{-6}.
\]

2 Assignments
All numbers below refer to the textbook: Calculus Concepts and Contexts, J. Stewart, 4th Ed.

- §8.5: 3, 8, 13, 18, 19, 23, 24, 27
- §8.6: 1, 2, 6, 8, 10, 12, 26, 28, 31, 37, 39, 40
- §8.7: 1, 6, 8, 10, 13, 14, 19, 22, 24, 27, 29, 30, 32, 34, 40, 41, 42, 45, 46, 48, 53, 59, 60, 62, 64, 66
- Extra credit §8.7: 69

Boldfaced problems weigh 2 points each and all other problems 1 point each. A random subset of the assigned problems with a total of 30 points will be graded. Additional extra credit of 10% will be given to you if you typeset your solutions in \LaTeX\ (ask me for the template tex file if you plan to do this). You might also get partial extra credit for typeset solutions of some problems. You will also get extra credit if you find typos or errors in the summary handouts.

Caution: please make sure that your handwriting is recognizable, otherwise you only get partial credit for the recognizable part.

Note: Taylor series is the most important math tool for engineers, therefore I have assigned correspondingly a fair number of problems. The next assignment will be much lighter.