To build a house foundation on a muddy river bed, the first two steps are (i) driving a number of pilings into the muddy bed firmly, and (ii) connecting these pilings by rails or plates to reinforce the whole structure. If the pilings in step (i) are not firm enough, or the connection between these pilings in step (ii) are not strong enough, the structure will be unstable, or even be washed away by the water flow.

Learning math is analogous. You need to completely understand individual definitions and theorems. The “firmness” of these pilings can be measured by the degree of clarity of your memorization. The “strength” of connections can be measured by how many proofs you know to navigate from one def/thm to another.

1 Overview

The main purposes of this competition are

• to give you a solid understanding on the process of efficient math-learning,
• to prepare you for exam 2,
• to give you an opportunity to make up for exam 1.

The competition is purely voluntary. However, if you choose to participate, your score will be added as extra credit to your score on exam 1.

(R1) One to three students will be selected as first-place winners; each of them will be given 18 points.

(R2) Two to five students will be selected as second-place winners; each of them will be given 15 points.

(R3) The extra credit given to other students varies between 0 to 12 points.

(R4) An extra perk to (R1): if the work of a first-place winner is perfect, s/he will receive a perfect score for exam 2, yet have the freedom of not taking it.

2 Specifications

To participate, you need to turn in

(I) a mind map for Chapter 9 and Chapter 10 out of the memorization sheet for Exam 2,

(II) proofs that connects definitions, theorems, formulas or corollaries in the mind map.

The handout of mind map for exam 1 provides an intuitive example of (I). However, in this competition, (I) must satisfy the following:

(M1) The mind map consists of rectangular boxes and connectors only.

(M2) A box captures a definition or formula or theorem or corollary and it contains

(b1) abbreviation of a def/frm/thm/cor, e.g. ‘D1’ refers to Definition 1 and ‘T17’ Theorem 17,

(b2) keywords or symbols for the main content,

(b3) an equation if there’s enough space.

(M3) A connector denotes the relationship between two boxes. There are four types of relationships:

(c1) \[ A \rightarrow B \] specialization. B is a special A.

(c2) \[ A \rightsquigarrow B \] composition. B has an object of A, i.e., the concept of A is employed to define the new concept of B.

(c3) \[ A \Rightarrow B \] implication. B can be deduced from A.

(c4) \[ A \leftrightarrow B \] equivalency. A ⇒ B and B ⇒ A.

(M4) All 36 def/frm/thm/cor in memorization sheet for exam 2 must show up on your mind map, otherwise you get no credit.

(II) are for the connectors of types (c3) and (c4); see the next section for an example.
3 An Example

Example 1.

D30: curvature $\Rightarrow$ T32: $\kappa = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$

Proof. D25 and F29 implies

$$r' = |r'| T = \frac{ds}{dt} T. \quad (1)$$

By the product rule (20c) in T22,

$$r'' = \frac{d^2s}{dt^2} T + \frac{ds}{dt} T'. \quad (2)$$

By D19 and T20, $(T \parallel T) \Rightarrow (T \times T = 0)$, hence

$$|r' \times r''| = \left(\frac{ds}{dt}\right)^2 |T \times T'|. \quad (3)$$

By D25 and D7, $|T| = 1$. Hence $T \perp T'$ by T27. Hence

$$|T \times T'| = |T'|$$

By D15. Then F29 and F31 imply

$$|r' \times r''| = |r'|^2 |T'| = \kappa(t) |r'|^3. \quad (4)$$

Dividing both sides with $|r'|^3$ completes the proof. $\square$

4 Hints

- Carefully think about the type of relationships between two boxes: most points of the mind map are about correct relationships.
- Arrange boxes in a rectangular array and avoid crossing of arrows as much as possible.
- You may add additional boxes for a better illustration of relationships. You may also break one def/thm into multiple boxes.
- For connectors of type (c1), put besides the arrow the conditions that specialize A to B.
- For connectors of type (c2), put besides the arrow other definitions or equation numbers that are also used in B.
- After finishing the mind map, go through numbers 1 to 36 to make sure they all show up.
- The more proofs you do, the more points you get.
- You get additional 25% credits if you typeset your work in \LaTeX. It will also give you an edge in the competition.