1. Find the complete solutions to the following systems, and use that solution to find a complete solution to the associated homogenous systems and a particular solution of the given systems. Write the solution of the homogenous systems as a linear combination of vectors:

a. \[
\begin{pmatrix}
3 & 4 & 1 & 2 \\
6 & 8 & 2 & 5 \\
9 & 12 & 3 & 10
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{pmatrix}
= 
\begin{pmatrix}
3 \\
7 \\
13
\end{pmatrix}.
\]

b. \[
\begin{pmatrix}
9 & -3 & 5 & 6 \\
6 & -2 & 3 & 1 \\
3 & -1 & 3 & 14
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{pmatrix}
= 
\begin{pmatrix}
4 \\
5 \\
-8
\end{pmatrix}.
\]

2. The equation \(x + y + z = 1\) can be viewed as a linear system of one equation in three unknowns. Express the general solution as a particular solution plus the complete solution of the homogenous problem. Interpret your answer geometrically.

3. The general solution of the homogeneous linear system

\[
A = \begin{pmatrix}
1 & 3 & -2 & 0 & 2 & 0 \\
2 & 6 & -5 & -2 & 4 & -3 \\
0 & 0 & 5 & 10 & 0 & 15 \\
2 & 6 & 0 & 8 & 4 & 18
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\]

is \(x_1 = -3r - 4s - 2t, x_2 = r, x_3 = -2s, x_4 = s, x_5 = t, x_6 = 0\).

a. Write \(X\) the general solution vector as a linear combination of 4 vectors. Verify that each of the 3 vectors in the general solution is perpendicular to each of the row vectors of the given matrix.

b. Given the number of free variables, describe the intersection of the 4 hyperplanes given by each of the 4 homogenous equations in \(x_1, \ldots, x_6\) (equivalent to the above matrix equation) sitting inside of 6-space.