1. Describe all conditions necessary for $A \in M_{m \times n}$ to be invertible.

2. List as many different ways as you can that will force a square matrix to be invertible.

3. Give all conditions necessary for a linear transformation $T : V \mapsto W$ to be invertible.

4. Give at least two different examples of a mappings from $M_{2 \times 3}$ to $P_2$ that are linear and two which are not linear.

5. Describe how to attach a matrix to a linear transformation $T : V \mapsto W$ given bases $B_V = \{v_1, \ldots, v_n\}$ and $B_W = \{w_1, \ldots, w_m\}$.

6. With the notation of the last exercise, if $[T]_{B_V, B_W} = A$ and $[v]_{B_V} = X$ with $X \in \mathbb{R}^n$ then how does one find the vector $w = T(v)$?

7. Given $A \in M_{m \times n}$ and RREF($A$) how does one find bases for COL($A$), ROW($A$), N($A$), and N($A^T$)?

8. With the notation of the last example find basis for $\mathbb{R}^n$ and a basis for $\mathbb{R}^m$ for which the matrix $A$ relative to the two bases is a partitioned matrix $\begin{pmatrix} B & 0 \\ 0 & 0 \end{pmatrix}$ where $B \in M_{k \times k}$ and $k = \text{Rk}(A)$?

9. With the notation of the last exercise, use the bases for Col($A$) and N($A^T$) to show how you find the orthogonal projection of $X \in \mathbb{R}^m$ onto Col($A$).

10. With the notation of the last exercise if $\text{Rk}(A) = k < m < n$ then describe how you find the least squares solution to $AX = Y$ if $Y \notin \text{Col}(A)$. Is there a unique solution in this case?