Worksheets for Math 3220 §2, Spring 2020

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In this CoViD-19 period of distance leaarning, the commentaries play the roles of daily lectures. Worksheets are daily assignments that give you the opportunity to show that you are participating. They are intended to relate to the the topic being discussed in the commentary, and should be easier than homework. Worksheets are due the day of the next class. Your worksheets will be scored, but as far as the grading goes, they will have weight zero in your final grade. For your convenience, the worksheet assignments are listed here.

March 20. (Postponed from Earthquake Wednesday.) Using Lagrange multipliers, find the highest and lowest points on the ellipse formed by the intersection of the cone $x^2 + y^2 = z^2$ and the plane $x + 2y + 3z = 3$.

March 23. Let $S = [0, 1] \times [0, 1]$ be the unit square in the plane. Find a bounded function $f : S \to \mathbb{R}$ which is not integrable and prove that your function is not integrable.

March 24. Let $T = [0, a] \times [0, b]$ be a rectangle in $\mathbb{R}^2$. Show that $g(x, y) = \sqrt{x + y}$ is integrable on T.

March 25. Let R be an aligned rectangle in $\mathbb{R}^d$. Show that if f is integrable on R then so is cf, where c is a constant. Show also that $\int_R cf = c \int_R f$.

March 27. Let $[0, 1] \times [0, 1]$ be a square in $\mathbb{R}^2$. For $E = [1, 1] \times [1, 1]$, show that χ_E is integrable on R and find $\int_R \chi_E$.

March 30. Let $E = \left\{ (\frac{1}{n}, \frac{1}{n}) : n \in \mathbb{Z} \right\} \cup \{(0, 0)\}$ be a set in $\mathbb{R}^2$. Show that E has volume zero.

April 3. Let $A, B \subset \mathbb{R}^d$ be Jordan Regions. Show that $(A \cup B) \backslash (A \cap B)$ is a Jordan region.

April 6. Let $f(x, y) = s(x+y)$ where $s(t)$ is the square wave and $A = \{(x, y) \in \mathbb{R}^2 : |x|+|y| \le 1\}$ is the diamond shaped region. Prove that $f(x, y)$ is integrable on A.

April 7. Find a bounded function $f(x, y) : S \times T \to \mathbb{R}$ where S and T are closed bounded intervals such that inequality holds

$$\underline{\int}_{S\times T} f(x, y)\, dV(x, y) < \underline{\int}_{S} f(x, y) \underline{\int}_{T} f(x, y)\, dV(y)\, dV(x).$$

April 8. Evaluate $\displaystyle\int_0^1 \int_0^1 \frac{xy^3}{(1 + x^2y^2)^2}\, dy\, dx.$

April 10. Find $\int_A y\, dV(x, y, z)$ where A is defined by the inequalities

$$0 \le x \le 1, \qquad 0 \le y \le 1 - x^2, \qquad 0 \le z \le x + y.$$

April 13. An orthogonal transformation is a linear map $A : \mathbb{R}^d \to \mathbb{R}^d$ that peserves the dot product, that is $Au \bullet Av - u \bullet v$ for all $u, v \in \mathbb{R}^d$. Prove that A preserves volume.

April 14. Find one place where the proof of Lemma 10.5.9 breaks down if there is no restriction on the aspect ratio.

April 15. Use the transformation $u = x - y$ and $v = 2x + 5y$ to evaluate the integral

$$I = \int_R x - y \, dV(x, y)$$

where R is the region in the first quadrant bounded by the curves $y = x$, $y = x - 3$, $2x + 5y = 10$ and $2x + 5y = 15$.

April 17. Let E be the trapezoid with vertices $(1, 1)$, $(2, 2)$, $(4, 0)$ and $(2, 0)$. Find

$$\int_E e^{(y-x)/(y+x)} \, dV(x, y).$$

April 20. Let U be the open region in the first quadrant between $x^2 + y^2 = 9$ and $x^2 + 2y^2 = 18$. make the change of variable $\varphi(x, y) = (x^2 + y^2, x^2 + 2y^2)$ to evaluate

$$\int_U xy(x^2 - y^2) \, dV(x, y).$$