Homework for Math 3220 §2, Fall 2019

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November 21, 2019

Our text is by Joseph L. Taylor, *Foundations of Analysis,* American Mathematical Soceity, Providence (2012). Please read the relevant sections in the text as well as any cited reference. Assignments are due the following Friday, or on April 24, whichever comes first.

Your written work reflects your professionalism. Make answers complete, self contained and written in good English. This means that you should copy or paraphrase each question, provide adequate explanation to help the reader understand the structure of your argument, be thorough in the details, state any theorem that you use and proofread your answer. If you use another source besides the text or the lecture notes, provide a citation.

Homework from Wednesday to Tuesday will be due Friday. Late homework that is up to one week late will receive half credit. Homework that is more than one week late will receive no credit at all. Homework that is placed in my mailbox in JWB 228 before 1:00 pm Friday afternoon will be considered to be on time.

Please hand in problems A1 on Friday, August 23.

A1. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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152[1, 3, 6, 10]
158[4, 5, 8]
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Please hand in problems B1 on Friday, August 30.

B1. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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167[5, 6, 12]
173[1, 8, 10]
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Please hand in problems C1 on Friday, September 6.

C1. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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178[1, 2, 4, 8, 10]
Problems from p. 182 postponed to next week.
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Please hand in problems D1 on Friday, September 13.

D1. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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182[1, 2, 4, 5, 8, 10, 11]
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Please hand in problems E1 on Friday, Sept. 20.

E1. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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188[6, 8, 10]
195[1, 5, 9,  12]
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Please hand in problems F1–F2 on Friday, Sept. 27.

F1. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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195[11]
201[5, 6, 10]
206[1, 7, 8]
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F2. Suppose $E \subset \mathbf{R}^p$. We say that E has the *Intermediate Value Property* if for every continuous function $f : E \to \mathbf{R}$ the set $f(E)$ is an interval. Show that E is connected if and only if it has the intermediate value property.

Please hand in problems G1 on Friday, Oct. 4.

G1. Please hand in the following exercises from Taylor's *Foundations of Analysis*. Read the review sections §8.4 and §8.5 about linear algebra. Do any problem whose solution isn't immediately clear.

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214[14],
221[9],
228[1, 9, 10],
235[3, 6, 10].
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Please hand in problems H1 and H2 on Friday, Oct. 18.

H1. Exercises from Taylor's *Foundations of Analysis*.

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241[3, 5, 10],
250[2, 7, 9].
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H2. (See 9.3[9]) Suppose that (x, y, z) are the Cartesian coordinates of a point in $\mathbf{R}^3$ and the spherical coordinates of the same point is given by

$$\begin{aligned} x &= r\cos\vartheta\,\sin\varphi, \\ y &= r\sin\vartheta\,\sin\varphi, \\ z &= r\cos\varphi. \end{aligned}$$

Let $u = f(x, y, z)$ be a $\mathcal{C}^2$ function on $\mathbf{R}^3$. Find a formula for the partial derivatives of u with respect to x, y, z in terms of partial derivatives with respect to r, ϑ, φ. Find a formula for the Laplacian of u in terms of partial derivatives with respect to r, ϑ, φ, where the Laplacian is given by

$$\Delta\, u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}.$$

Please hand in problems I1 and I2 on Friday, Oct. 25.

I1. Exercises from Taylor's *Foundations of Analysis.*

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259[2, 3, 6, 7, 11].
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I2. Find the critical point (s_0, t_0) in the set $\{(s,t) \in \mathbb{R}^2 : s > 0\}$ for the function with any real A and $B > 0$,

$$f(s,t) = \log(s) + \frac{(t-A)^2 + B^2}{s}.$$

Find the second order Taylor's expansion for f about the point (s_0, t_0). Prove that f has a local minimum at (s_0, t_0).

Please hand in problems J1 on Friday, Nov. 1.

J1. Exercises from Taylor's *Foundations of Analysis.*

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265[2, 9, 10, 11],
272[1, 3, 5, 10].
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Please hand in problems K1 – K2 on Friday, Nov. 8.

K1. Exercises from Taylor's *Foundations of Analysis.*

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259[13],
282[5, 6, 9],
287[2, 4, 12].
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K2. Find all extrema of the function $f(x) = x_1^2 + \cdots + x_n^2$ subject to the constraint $|x_1|^p + \cdots + |x_n|^p = 1$. If $1 \le p \le 2$ show for any x and n that

$$n^{\frac{p-2}{2p}} \left(|x_1|^p + \cdots + |x_n|^p\right)^{\frac{1}{p}} \le \sqrt{x_1^2 + \cdots + x_n^2} \le \left(|x_1|^p + \cdots + |x_n|^p\right)^{\frac{1}{p}}.$$

Please hand in problems L1 on Friday, Nov. 15.

L1. Exercises from Taylor's *Foundations of Analysis.*

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293[1, 5, 9, 14],
Problems from section 10.4 postponed to next week.
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Please hand in problems M1 on Friday, Nov. 22.

M1. Exercises from Taylor's *Foundations of Analysis.*

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302[2, 7, 9, 10].
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Please hand in problems N1 on Monday, Dec. 2.

N1. Exercises from Taylor's *Foundations of Analysis*.

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314[2, 3, 4, 6, 7, 12].
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The FINAL EXAM is Thurs., Dec. 12 at 8:00 AM in the usual classroom, ST 216.