Homework for Math 3220 §2, Spring 2018

A. Treibergs, Instructor

April 11, 2018

Our text is by Joseph L. Taylor, *Foundations of Analysis,* American Mathematical Soceity, Providence (2012). Please read the relevant sections in the text as well as any cited reference. Assignments are due the following Friday, or on April 24, whichever comes first.

Your written work reflects your professionalism. Make answers complete, self contained and written in good English. This means that you should copy or paraphrase each question, provide adequate explanation to help the reader understand the structure of your argument, be thorough in the details, state any theorem that you use and proofread your answer.

Homework from Wednesday to Tuesday will be due Friday. Late homework that is up to one week late will receive half credit. Homework that is more than one week late will receive no credit at all. The homework reader is Brendan Black. Homework that is placed in his mailbox in JWB 228 before he picks it up not later than 3:00 pm Friday afternoon will be considered to be on time.

Please hand in problems A1 on Friday, January 12.

A1. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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152[6, 7, 8, 12]
158[3, 7, 11]
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Please hand in problems B1 on Friday, January 19.

B1. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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167[4, 5, 6, 12]
173[1, 8, 11]
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Please hand in problems C1 on Friday, January 26.

C1. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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173[14, 15]
178[1, 2, 8, 10]
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Please hand in problems D1 on Friday, Feb. 2.

D1. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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182[1, 2, 4, 8, 10, 11]
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Please hand in problems E1 on Friday, Feb. 9.

E1. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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183[5]
188[2, 6, 10, 11]
195[1, 5]
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Please hand in problems F1 on Friday, Feb. 16.

F1. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

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195[11]
201[5, 6, 7, 10, 11]
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Please hand in problems G1 on Friday, Feb. 23.

G1. Please hand in the following exercises from Taylor's *Foundations of Analysis.* Read the review sections §8.3 and §8.4 about linear algebra. Do any problem whose solution isn't immediately clear.

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206[1, 7, 8, 10],
214[14],
221[10].
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Please hand in problems H1 on Friday, Mar. 2.

H1. Exercises from Taylor's *Foundations of Analysis.*

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228[1, 8, 9, 10].
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Please hand in problems I1 and I2 on Friday, Mar. 9.

I1. Exercises from Taylor's *Foundations of Analysis.*

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235[3, 6, 10],
241[3, 5, 10]
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I2. (See 9.3[9]) Suppose that (x, y, z) are the Cartesian coordinates of a point in $\mathbf{R}^3$ and the spherical coordinates of the same point is given by

$$\begin{aligned} x &= r\cos\vartheta\,\sin\varphi, \\ y &= r\sin\vartheta\,\sin\varphi, \\ z &= r\cos\varphi. \end{aligned}$$

Let $u = f(x, y, z)$ be a $\mathcal{C}^2$ function on $\mathbf{R}^3$. Find a formula for the partial derivatives of u with respect to x, y, z in terms of partial derivatives with respect to r, ϑ, φ. Find a formula for the Laplacian of u in terms of partial derivatives with respect to r, ϑ, φ, where the Laplacian is given by

$$\Delta\, u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}.$$

Please hand in problems J1 and J2 on Friday, Mar. 16.

J1. Exercises from Taylor's *Foundations of Analysis.*

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250[2, 8, 12],
259[3, 6, 9].
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J2. Find the critical point (s_0, t_0) in the set $\{(s, t) \in \mathbb{R}^2 : s > 0\}$ for the function with any real A and $B > 0$,

$$f(s, t) = \log(s) + \frac{(t - A)^2 + B^2}{s}.$$

Find the second order Taylor's expansion for f about the point (s_0, t_0). Prove that f has a local minimum at (s_0, t_0).

Please hand in problems K1 – K3 on Friday, Mar. 30.

K1. Exercises from Taylor's *Foundations of Analysis.*

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259[13],
265[2, 8].
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K2. Find all extrema of the function $f(x) = x_1^2 + \cdots + x_n^2$ subject to the constraint $|x_1|^p + \cdots + |x_n|^p = 1$. If $1 \le p \le 2$ show for any x and n that

$$n^{\frac{p-2}{2p}} \left(|x_1|^p + \cdots + |x_n|^p\right)^{\frac{1}{p}} \le \sqrt{x_1^2 + \cdots + x_n^2} \le \left(|x_1|^p + \cdots + |x_n|^p\right)^{\frac{1}{p}}.$$

K3. Let $F : \mathbf{R}^2 \to \mathbf{R}^2$ be given by

$$\begin{aligned} x &= u^2 - v^2, \\ y &= 2uv; \end{aligned}$$

Find an open set $U \subset \mathbf{R}^2$ such that $(3, 4) \in U$ and $V = F(U)$ is an open set for which there is a $\mathcal{C}^1$ function $G : V \to U$ such that

$$G \circ F(u, v) = (u, v) \quad \text{for all } (u, v) \in U \text{ and } \quad F \circ G(x, y) = (x, y) \quad \text{for all } (x, y) \in V.$$

Find the differential $dG(F(3, 4))$.

Please hand in problems L1 on Friday, Apr. 6.

L1. Exercises from Taylor's *Foundations of Analysis*.

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265[9, 10, 11],
272[3, 10].
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Please hand in problems M1 and M2 on Friday, Apr. 13.

M1. Exercises from Taylor's *Foundations of Analysis*.

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272[1, 5, 8],
282[5, 6, 9].
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M2. In section §9.7 the Implicit Function Theorem was deduced from the Inverse Function Theorem. Show that the Inverse Function Theorem can be deduced from the Implicit Function Theorem.

Please hand in problems N1 on Friday, Apr. 20.

N1. Exercises from Taylor's *Foundations of Analysis*.

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287[2, 4, 12],
293[5, 9],
302[2, 7, 9],
413[2, 7].
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The FINAL EXAM is Thurs., April 26 at 10:30 AM in the usual classroom, LCB 225.